LABORATORY EVALUATION OF THE RESPONSE OF REINFORCED CONCRETE TO INTERNAL VIBRATION

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BY

O.T. OLATEJU

JHRP

JOINT HIGHWAY RESEARCH PROJECT
PURDUE UNIVERSITY AND
INDIANA STATE HIGHWAY COMMISSION

Final Report

LABORATORY EVALUATION OF THE RESPONSE OF REINFORCED CONCRETE TO INTERNAL VIBRATION

TO: J. F. McLaughlin, Director
Joint Highway Research Project

December 13, 1973

Project: C-36-67G

FROM: H. L. Michael, Associate Director
Joint Highway Research Project

File: 9-11-7

Attached is the Final Report on Phase I of the HPR Part II Research Study titled "Compaction of Reinforced Concrete by Internal Vibration". The Report is titled "Laboratory Evaluation of the Response of Reinforced Concrete to Internal Vibration" and has been authored by Mr. O. T. Olateju, Graduate Instructor in Research on our staff. Professors C. F. Scholer and J. A. Havers directed the research. Mr. Olateju also used the report as his thesis for the Ph.D. degree from Purdue University.

The objectives of this Phase I of the Study were to evaluate the effectiveness of various vibratory effects on reinforced concrete and to determine which parameters most affect the compaction of reinforced concrete in slipform paving. Three vibratory frequencies were found to be equally effective and several other results relative to parameters affecting compaction were found. Recommendations for slump range, frequency and amplitude range of vibration and additional research are made.

A Phase II of the Study is planned and will be submitted as a proposal for continuation shortly.

The attached Report is submitted for acceptance as fulfillment of the objectives of Phase I of this Study.

Respectfully submitted,

Harold L Michael

Harold L. Michael
Associate Director

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16. Abstract
The objectives of this first phase of a proposed two-phase study were 1) to evaluate the effectiveness of various vibratory efforts on several reinforced concrete specimens, and 2) to determine which parameters most affect the compaction of reinforced concrete in slipform paving. To achieve these objectives, a statistically designed experiment was performed. In this experiment, three different mix designs and nine different combinations of frequency and duration of vibration (treatments) were examined. The factors measured included the frequency and amplitude of vibration, particle acceleration in the concrete, pulse velocity, nuclear density and several pertinent concrete parameters. The data obtained were statistically analyzed by computer and the results indicated that the 13,200, 12,000, and 10,800 cycles per minute (cpm) frequencies were at 5% level of significance equally effective in compacting concrete and that all the three frequencies were significantly more effective than the 9,600 and the 7,300 cpm frequencies. Other results were that the measured frequency in concrete was about 15% lower than the frequency input at the vibrator; that the mean density below steel was about 6.5% lower than the density above steel; and that consistency (slump) was a very important factor in the vibratory compaction of concrete. These results were followed by the recommendations that 1) a slump range between 1 and 2-1/4 inches should be adopte for slipform paving; 2) that a frequency range between 10,800 and 12,000 cpm, and an amplitude range between 0.0045 and 0.0055 inches should also be adopted; and 3) that studies should be initiated to determine the reason for the reduction of the input frequency in concrete.

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Final Report - Phase I

LABORATORY EVALUATION OF THE RESPONSE OF REINFORCED CONCRETE TO INTERNAL VIBRATION

by

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and the

U. S. Department of Transportation
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The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

Purdue University
West Lafayette, Indiana

December 13, 1973

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TABLE OF CONTENTS

Page

LIST OF TABLES vii

LIST OF FIGURES viii

ABSTRACT x

INTRODUCTION 1

REVIEW OF LITERATURE 7

  Theoretical Investigations 7

    Stages of Compaction 8

    Attenuation of Vibratory Energy 10

  Laboratory Investigations 14

    Studies of Attenuation of Vibration 15

    Studies of Frequency 22

    Studies Involving Surface Vibration 23

    Studies Involving Reinforcement 24

  Investigations in Various States 24

  General Comments 27

  Characterization of Concrete Material 28

DESGN OF EXPERIMENT 33

  Plan of Experiment 33

  Composite Statistical Design for Experimental Work 35

  Subsequent Modification to Composite Statistical Design 40

TABLE OF CONTENTS, cont.

Page

APPARATUS AND EQUIPMENT 41

General Apparatus 41

Forms 41

Vibrators 42

Generator 44

V-Scope Apparatus 44

Nuclear Density Gauge 44

Instrumentation 51

Accelerometers 52

Oscillographic Recorder 56

Linear Variable Differential Transformer (LVDT) 57

LARR-V Digital Coordinatograph 58

DATA COLLECTION AND PROCESSING 60

Procedure for Data Collection 66

Preparation of Specimens 66

Measurement of Nuclear Density and Pulse Velocity .. 69

Measurement of Amplitude 73

Comments on Testing Program 79

Data Analysis 79

Reduction of Data 79

Statistical Analysis 81

Analysis of Nuclear Density Data 82

Analysis of Pulse Velocity Data 86

Other Analyses 87

TABLE OF CONTENTS, cont.

Page

RESULTS OF DATA ANALYSIS 90

Results from Analysis of Variance 90

Results from Newman-Keul's Test 91

Results from Regression Analysis 93

Summary of Results 96

DISCUSSION OF RESULTS 99

RECOMMENDATIONS 108

LIST OF REFERENCES 110

General References 113

APPENDICES

Appendix A: Gradation Curves for Aggregates 115

Appendix B: Computer Program for the Analysis of Nuclear Density Data 117

Appendix C: Computer Program for the Analysis of Pulse Velocity Data 124

Appendix D: Computer Program for the Analysis of Acceleration and Displacement Data 128

Appendix E: Data Summary Program 158

Appendix F: Output for Nuclear Density from ANOVA Program (BMD8V) 165

Appendix G: Copies of ANOVA Programs Used 166

Appendix H: Sample Calculation for Pooling 191

Appendix J: Computer Program and Typical Printout for Newman-Keul's Test 192

Appendix K: Typical Printout from Regression Analysis Program "SPSS 15" 201

VITA 204

LIST OF TABLES

Table Page

1. Treatment Combinations Used in Composite Statistical Design 37

2. Randomization of Mix Designs 37

3. Order of Carrying Out Tests on Experimental Units 39

4. Gradation of Each Delivery of Coarse and Fine Aggregates 62

5. Basic Components of Each Mix Design Used 65

6. Arranged Computer Printout for Split-Split Plot Design 84

7. Summary of ANOVA Results 88

8. Rank of Means by Newman-Keul's Test for 9 Treatment Combinations for All Mixes 92

9. Rank of Means by Newman-Keul's Test for 5 Frequencies for All Mixes 92

10. Differences Between Nuclear Density Above and Below Steel in Percent 98

11. Input and Response Frequencies for All Mixes and the Differences 101

12. Variation of Input Amplitude with the Response Amplitude 103

13. Slumps and Corresponding Theoretical Input Treatments for the Specimens Tested 103

LIST OF FIGURES

Figure		Page
1.	Cores Taken from Prematurely Deteriorated Pavement Segments on Indiana Highways	3
2.	Various Regional Specifications for Internal and Surface Vibrators	4
3.	Venkatramiah's Concrete Compaction Curve	9
4.	Bergstrom's Concrete Deformability Curve	9
5.	Pigman and Hornibrook's Decay of Amplitude Curves ...	16
6.	Las Forssblad's Decay of Acceleration Curve	17
7.	Bergstrom's Decay of Amplitude of Pressure and Displacement Curves	18
8.	Las Forrsblad's Curves for the Relationship Between Radius of Action, Frequency and Amplitude ...	20
9.	Power Input of an Internal Vibrator During a Single Immersion in Concrete	21
10.	Variations of Density with Depth of Compaction	25
11.	Effect of Reinforcement on Depth of Compaction of Concrete	26
12.	Relationship Between Shear Stress and Rate of Deformation for Newtonian and Non-Newtonian Fluids	30
13.	Nessim and Wadja's Rheological Curves for Cement Paste	32
14.	Factor Space Coverage of Experimental Variables	38
15.	Details of the Forms Used	43
16.	Details of the Vibrator and Its Associated Parts	45
17.	Basic Operation Details of the V-Scope	46

LIST OF FIGURES, cont.

Figure Page

18. Nuclear Chicago's Density Probe 48

19. Typical Calibration Data Sheet for Nuclear Gauges 50

20. Schematic of the Positions of the Three Accelerometers in the Form 53

21. Locations of Accelerometers Relative to the Vibrator within the Form 54

22. Schematic of Accelerometers 55

23. The LARR-V Digital Coordinatograph 59

24. Typical Specimen with Accelerometers before Vibration and Companion Specimens after Vibration 70

25. The Lowered Surcharge Portion of the Form and Some Finished Specimens 71

26. Positions of Density Readings on the Specimens 72

27. Positions of Pulse Velocity Readings on the Specimens 74

28. Triaxial Mounting of Accelerometers 75

29. Schematic of LVDT Connection to Vibrator 77

30. LVDT Attachment to Vibrator at Position One 78

31. Typical Acceleration and Displacement Records 80

ABSTRACT

Ọlatẹju, Olubayọ Timothy. Ph.D., Purdue University, December 1973.
Laboratory Evaluation of the Response of Reinforced Concrete to Internal Vibration. Major Professors: Dr. John Havers and
Dr. Charles Scholer.

This project is a part of the National Experimental Evaluation Program for concrete compaction by internal vibration and constitutes the first phase of a proposed two-part study. The objectives of this first phase are to evaluate the effectiveness of various vibratory efforts on several reinforced concrete specimens made from a number of different mix designs, and to determine which parameters most affect concrete compaction in slipform paving.

An extensive review of the pertinent literature preceded the statistically designed, performed and analyzed laboratory experiment which is reported herein. The literature review examined the results of various theoretical and experimental investigations that have been carried out in the United States and other parts of the world. It also examined some results of the recent national cooperative effort within the United States to investigate concrete vibration. Based in part on this literature review, concrete in its plastic state was viewed for study purposes as a non-homogeneous thixotropic fluid with large masses suspended in it.

The statistical design for the experimental study was a two-factor factorial plus a four-factor split plot, split-split plot. This

composite design was appropriate because of the number of factors involved, the desired inference space, and the various restrictions on the randomization of the factors involved in the experiment. However, it had to be modified during the execution stage because of some unfortunate circumstances which were beyond the control of the experimenter.

Three different mix designs were investigated. Two were made from crushed limestone and the third from gravel aggregate. Nine different combinations of frequency and duration of vibration (treatments) were investigated. The frequencies were varied between 7,200 and 13,200 cycles per minute (cpm), and the durations between 5 and 13 seconds. All of the specimens tested were reinforced with No. 5 bars spaced 6 inches apart, and each had an eight inch thick 'pavement segment' overlaid by a four inch concrete surcharge which was screeded off after vibration. The vibratory effort was evaluated by measuring the particle acceleration during vibration, and the density and pulse velocity after vibration. The data analysis utilized simple computer programs developed by the researcher and several computerized statistical programs available at Purdue University.

The experimental results indicated that a frequency of 12,000 cpm applied for 7 seconds produced the highest mean density and pulse velocity for all the mix designs tested. However, when statistical analyses were performed, it was found that the 12,000 cpm frequency was not significantly more effective in compacting concrete than the 10,800 or the 13,200 cpm at 5% level of

significance. All three of these frequencies were, however, significantly more effective than either the 9,600 or the 7,200 cpm frequency. It was also found that the frequency measured in the concrete with accelerometers was lower than the frequency input of the vibrator by more than 15%, that the density of concrete above the steel was about 6.5% higher than that below steel, and that the compaction of concrete was not as dependent on frequency and amplitude as had previously been conjectured.

The results of this experiment suggested the following recommendations:

1. A slump range of between 1 and 2¼ inches is necessary for proper compaction by vibration to be achieved. This slump range should therefore be adopted.
2. A frequency range between 10,800 and 12,000 cpm, and an amplitude range between 0.0045 and 0.0055 inches should be applied to concrete with a slump range between the recommended 1 and 2¼ inches in slipform paving.
3. Instruments to indicate the frequency of vibration to the slipform paver operator and those to non-destructively measure the compaction of concrete at depths more than 3 inches below the surface are needed.
4. Investigation to determine what is happening at the interface of concrete and vibrator should be initiated since it may help explain the phenomenon behind the reduction of the input frequency in concrete.

INTRODUCTION

The first half of this century has witnessed a rapid development in the practices related to the manufacture, placement and compaction of concrete. However, the advance in our knowledge of the material itself and of the mechanism of its dynamic properties lags behind modern requirements. This anomaly has led to inefficient and uneconomical use of this remarkably deformable and versatile structural material (1)[1].

The compaction of concrete is a process whereby the void spaces between aggregate particles are reduced. In so doing, the particles of the aggregate are constrained to pack more closely together. The usual goal is to approach the maximum potential density of the concrete and thereby approach its maximum compressive strength (2). Although the compaction of concrete can be achieved in various ways, it has long been recognized that mechanical aids are required to compact dry harsh mixes, especially when these mixes are placed around reinforcement or other obstructions. The most successful method in these circumstances has been the use of the mechanical vibrator.

---

[1] Numbers in parentheses indicate reference numbers in the list of references.

Concrete vibrators have been used for many years in this country to place highway pavements. However, most of our knowledge concerning concrete vibrators has been obtained from intuitive practices rather than controlled programs of research. Inadequate concrete consolidation has led to premature deterioration and eventual retirement of several structures, especially highway pavements (see Figure 1).

Several laboratory and field experiments have been conducted in the United States and many other parts of the world and significant advancements have been made within the last two decades in attempts to advance our knowledge of the vibration of plastic concrete. However, laboratory and field investigations have often yielded results which are difficult to interpret or too general to apply in practice. There appear to be major deficiencies in our quantitative understanding of the degree of compaction that can be achieved in a given concrete mix by a given vibrating element which is operating under a stated set of conditions. This lack of knowledge is evidenced by the various regional specifications for internal and surface vibrators (see Figure 2). By way of illustration, the specifications of the Indiana State Highway Commission read as follows (3):

> ". . . . The frequency of the surface vibrators shall not be less than 3,500 impulses per minute and the frequency of the internal type not less than 5,000 for tube vibrators and 7,000 for spud vibrators unless modified by the engineer . . ."

The comparable specification of the U.S. Army Corps of Engineers reads (4):

> "Internal vibrators shall maintain a frequency when submerged in concrete of not less than 6,000 impulses per minute for spuds with diameters greater than five inches and 7,000 impulses for smaller spuds. The intensity (amplitude) of vibration shall be sufficient to produce satisfactory

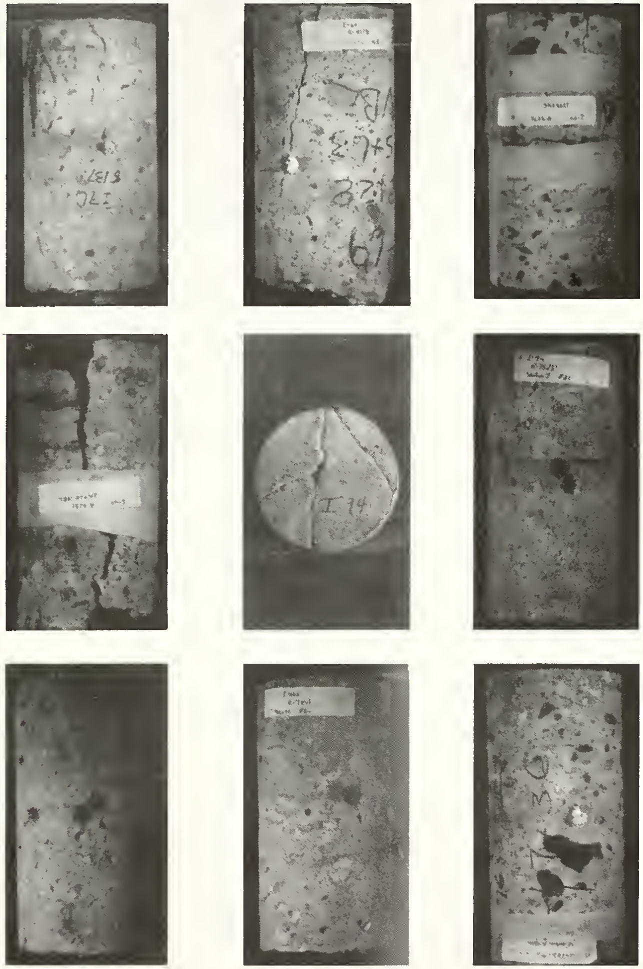

Figure 1

Cores Taken from Prematurely Deteriorated Pavement Segments on Indiana Highways

		Minimum Frequency (impulses per minute)			
		Surface	Internal (Tube)	Internal (Spud)	Adjacent to Forms
1	CONNECTICUT	--	3,500	3,500	--
	MAINE	3,500	5,000	7,000	3,500
	MASSACHUSETTS	3.500	5.000	7.000	3,500
	NEW HAMPSHIRE	P. C. C. Pavement covered by Special Provision			
	NEW JERSEY	5.000	5.000	5.000	5,000
	NEW YORK	4,000	4.000	4 000	4.000
	RHODE ISLAND	3,500	5,000	7,000	3.500
	VERMONT	--	4,500	4,500	--
	PUERTO RICO	--	5.000	5.000	--
2	DELAWARE	Consolidation by Machine Methods			
	MARYLAND	--	--	--	--
	OHIO	--	5 000	7,000	--
	PENNSYLVANIA	--	6,000	6,000	--
	VIRGINIA	3.500	5,000	7,000	3,500
	WEST VIRGINIA	3.500	5 000	5.000 - 10,000	3,500
	DIST. OF COL.	3.500	--	5.000	--
3	ALABAMA	3,500	5,000	7,000	3,500
	FLORIDA	3,500	3 500	3.500	3,500
	GEORGIA	3.500	3 500	3 500	3.500
	MISSISSIPPI	3,500	5,000	7,000	3,500
	NORTH CAROLINA	3.500	3.500	3.500	3 500
	SOUTH CAROLINA	3.500	5,000	7,000	3 500
	TENNESSEE	Frequency recommended to manufacturer.			
4	ILLINOIS	3,500	7.000	7,000	--
	INDIANA	3,500	5,000	7.000	--
	KENTUCKY	3,600	3,600	3,600	3,600
	MICHIGAN	--	5,000	5,000	--
	WISCONSIN	3,500	7,000	7,000	
5	IOWA	3,500	3,500	3,500	3,500
	KANSAS	Vibration required, but no frequency specified.			
	MINNESOTA	3,600	6,000	6,000	3.600
	MISSOURI	3,600	4,500	4,500	--
	NEBRASKA	4,000	4,000	4,000	4,000
	NORTH DAKOTA	Internal and Surface - Frequency such as not to segregate.			
	SOUTH DAKOTA	3,500	5,000	7,000	3,500
6	ARKANSAS	3,500	3,500	3,500	3,500
	LOUISIANA	3,500	5,000	7,000	--
	OKLAHOMA	3,500	5,000	7,000	--
	TEXAS	Vibration required - Frequency as determined by Engineer.			
7	ARIZONA	3,500	3.500	3,500	3.500
	CALIFORNIA	3,500	5.000	5 000	--
	NEVADA	3,500	5,000	5,000	--
	HAWAII	3,500	5,000	7,000	--
8	ALASKA	3.500	5.000	7.000	3,500
	IDAHO	Suitable vibrator.			
	MONTANA	Vibration required - Frequency not specified.			
	OREGON	3.500	3 500	3 500	3.500
	WASHINGTON	--	5,000	5.000	--
9	COLORADO	3.500	5.000	7,000	--
	NEW MEXICO	Vibration required - Frequency not specified.			
	UTAH	--	--	--	--
	WYOMING	3.500	5,000	7,000	3,500
	AASHO	3,500	5,000	7,000	3 500

Figure 2

Various Regional Specifications for Internal and Surface Vibrators

Federal Highways Admin

> consolidation. The duration of vibration shall be limited to that necessary to produce satisfactory consolidation".

These two specifications leave much to the discretion of the 'engineer' or whoever decides on the amount of vibration that produces "satisfactory consolidation". Modification by the engineer until "satisfactory consolidation" has been achieved is obviously not a quantitative definition of the desired compaction.

Engineers and contractors throughout the country have suggested that pavements are either not getting optimum compaction, or that the mix ingredients may be segregating during vibration. Many field problems such as cold joints, honey-combing at the base of pavements, bumps and dips on pavement surfaces and at construction joints, and early deterioration of pavements have been attributed to improper vibration. There are other factors that could have caused or contributed to such failures, but vibration is an aspect of construction which is insufficiently understood.

The growing acceptance of slipform pavers which employ both surface and internal vibrations has contributed to the current need for quantitative knowledge concerning concrete vibration. In fact, concrete vibration during slipform paving became of such concern in Iowa that the use of spud vibrators was banned until further studies could be made (5).

Research programs which contribute to our understanding of the mechanics of concrete vibration have been assigned a high priority by several investigating organizations, such as the Highway Research Board, Federal Highways Administration, Portland Cement Association, American Concrete Paving Association, American Concrete Institute,

many state highway departments and vibration equipment manufacturers.

The foregoing discussion strongly suggests the need for research which will improve our understanding of vibration effects on concrete compaction. It should not be inferred from this statement that extensive research has not already been conducted into the mechanism of concrete vibration; however, little use has been made of the results of most of these investigations. This is due in part to the conflicting conclusions of the research studies, and also to the restrictions which were placed on the experimental programs and hence on the conclusions drawn therefrom.

The objectives of this study reported herein are to determine which vibrator and concrete parameters affect the compaction of reinforced concrete specimens, and to evaluate the effectiveness of various vibratory efforts on several concrete mixes in producing maximum density. The method of achieving these goals is to examine the responses of various concrete mixes to different levels of vibration and to determine the statistical significance of the observed results.

This report starts with a selective review of prior investigations. The details of the experiment are then described, including the design, collection and analysis of data, and the results of the various analyses are presented. The report concludes with a discussion of the results and recommendations.

REVIEW OF LITERATURE

The objective of earlier research programs has been to better understand what happens to concrete when it is vibrated. The research approaches have been both theoretical and experimental. Attempts have been made to simulate conditions which actually occur in the field by using vibrating tables, molds which were vibrated at several frequencies and amplitudes, and steel and wooden forms of various shapes and sizes into which vibrating elements have been inserted or attached. This review concetrates on the results of the various investigations rather than on the methods employed in executing them.

Theoretical Investigations

The behavior of concrete under vibration is undoubtedly a complex process. It involves wave propagation in a non-uniform medium which, since it has thixotropic properties, changes its state with time (6). The problem is further complicated by a multiplicity of parameters relating to the vibrator, the concrete, the container in which the concrete is being vibrated, and the reinforcement, if the concrete is reinforced. Because of these complexities, most of the initial studies of vibration consisted of theoretical efforts to explain what happens when concrete is vibrated. These explanations have varied from the identification of various stages

of compaction to the manipulation of Newton's basic equation which expresses the Second Law of Motion.

Stages of Compaction

In their separate attempts to explain concrete vibration phenomena, Stewart and Venkatramiah (6, 7) identified three different phases or stages of compaction of concrete during vibration. The first phase was characterized by the initial collapse of the deposited concrete and the subsequent transformation of the mass into a viscous fluid. The second phase was one of de-aeration in which entrapped air bubbles were forced upwards and the voids were substantially reduced. The same stages were recognized by Kolek (8). In the third stage, according to Stewart and Venkatramiah, there was further compaction of the mass and occasionally a small bubble of air escaped. If the vibration was prolonged after this stage, segregation might follow. The first two phases occured within the first 10 seconds of vibration (see Figure 3).

These three stages were also recognized by Bergstrom (9) and were described in his concrete deformability treatise. Bergstrom's deformability curve, reproduced in Figure 4, includes three stages within the "compaction period". The three phases are not independent of each other, but they provide a common basis whereby researchers can identify significant parameters and study their influences on concrete compaction.

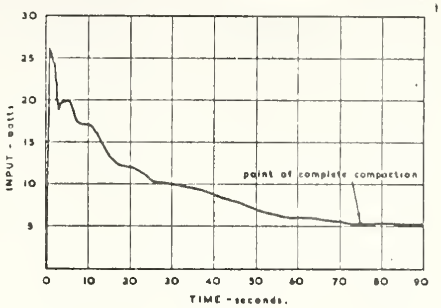

Figure 3

Venkatramiah's Concrete Compaction Curve

(ref. 7)

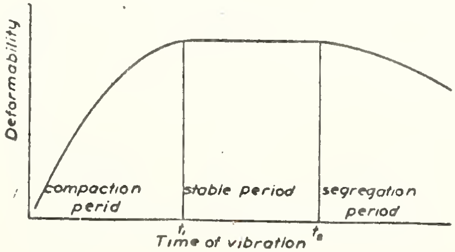

Figure 4

Bergstrom's Concrete Deformability Curve

(ref. 9)

Attenuation of Vibratory Energy

A vibrator transmits its energy to the concrete in the form of waves which are rapidly attenuated as they move outward from the forcing source. Las Forssblad (10) attributed this decay to two causes, namely, a reduction in the energy flux density due to the outward propagation of the vibratory waves, and the absorption of energy by the material. Farrar and Sullivan (11, 12) relate it to the "impedance theory" postulated by P. F. Chenea (13) on general vibratory systems. This theory represents the vibratory system as a combination of elastic, viscous and inertial elements and proceeds to examine the effects of each such element upon the behavior of the material system under consideration. Sullivan went one step further by relating the impedance theory to mechanical impedance, a complex function defined as the ratio of the exciting force to the displacement which the force produces. From these postulations, efforts were made to obtain the rates of energy damping by the application of basic vibratory equation.

The basic equation for the impressed force due to an unbalance is taken as:

$$\begin{aligned} M\ddot{x} + c\dot{x} + kx &= F_o \cos wt \\ &= um'ew^2t \qquad (14) \end{aligned}$$

where:

x = amplitude of vibration

u = m'/M

m' = mass of the eccentric

M = total mass of the system

F_o = maximum value of an impressed unbalanced force

w = circular frequency at which force is applied

c = damping coefficient which in concrete can be due to viscous damping brought about by the mix consistency

k = 'spring' constant for the concrete

e = eccentricity of the mass, m'

The forcing function is harmonic and the solution of the equation involves complex numbers to obtain the value of the amplitude of vibration, x. On obtaining the solution, it then becomes necessary to determine the constants k and c. Several equations have been proposed to establish the rate of attenuation of vibratory waves; a few of them are given below.

Dessoff (10) deduced the expression

$$S_2 = S_1 \sqrt{\frac{R_1}{R_2}} \left(e^{-\left(\Omega/2 \left(R_2 - R_1\right)\right)} \right)$$

where S_1, S_2 are amplitudes at the respective distances R_1 and R_2 from the center line of the internal vibrator; Ω represents the damping coefficient; the expression $\sqrt{\frac{R_1}{R_2}}$ represents the reduction in energy density which is proportional to the square of the amplitude; and the exponential represents the energy absorption.

Kolek (8) stated the equation for amplitude after concrete had undergone considerable amount of compaction as:

$$A = A_o W \sqrt{\frac{d}{k}}\ e^{-\left(\frac{Bx}{2}\right)}$$

where:

A_o = amplitude of the source

W = angular frequency

d = mean density of the medium

k = bulk density of the medium

B = absorption coefficient

x = distance from source (8)

Suguichi (4) has observed that the equation proposed by Kolek is dimensionally unbalanced.

L'Hermite (4) assumed that the damping coefficient increased linearly as the frequency and derived an amplitude on this basis as:

$$A = A_o e^{-\left(\frac{\alpha \sigma_o \nu x}{2}\right)}$$

where:

A_o = same as above

α = arbitrary constant

σ_o = initial coefficient of void space

ν = frequency of vibration

x = distance from source

Pigman and Hornibrook in their studies (15) concluded that L'Hermite's assumption of a linearly-increasing damping coefficient was not valid.

Finally, Farrar (11) postulated that the theoretical decay of vibration could be stated as:

$$A = A_o \cosh Nx$$

where:

A_o = acceleration amplitude at any given reference point, usually the origin of the vibration

A = acceleration amplitude at a point distance x from the point of A_o

N = constant depending upon concrete mix

x = distance between the two points

Farrar found an experimental value of N = 0.06 for the one mix and frequency (3,000 cpm)[2] that he used, and suggested that experimental values of N could similarly be determined for other mixes. All of the formulas just described have assumed an exponential decay, since $\cosh x = \frac{1}{2}(e^{x} + e^{-x})$. Dessoff and L'Hermite (10) have verified that the variation of acceleration with distance from the vibrator obeys this assumed exponential decay at least up to 7,000 cpm of vibration.

Although the amplitude of vibration had long been related to the amplitude of the waves generated in the concrete, Kolek (8) showed that the two amplitudes were also dependent on the frequency of vibration. It then becomes important to find out which parameters of vibration, namely, frequency, amplitude, acceleration, or radius of action of the vibrator actually influence the compaction of concrete. Another factor to be considered is the interaction between these parameters and the concrete parameters, such as the consistency of the mix.

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[2] cpm is cycles per minute

Laboratory Investigations

Experiments by the various authors mentioned in the preceding pages and by other people interested in the phenomenon of concrete vibration have sought either to prove or to disprove the theories propounded, or to establish completely different courses of action from those previously mentioned. These courses of action have involved more parameters than those identified by the impedance theory.

There are numerous parameters involved in concrete vibration and a complete investigation of them would require an extensive series of tests. The experimental studies carried out so far have been confined to limited ranges of selected vibrator or concrete parameters, seeking to obtain bases for comparisons. The theoretical studies, as previously noted, have postulated an exponential form of decay in the wave which advances within the concrete mass. Parameters identified in these theories include the amplitude and frequency of vibration. The acceleration of a particle within the vibrated mass is believed to be a function of the amplitude of the waves. Therefore, many investigators have studied the decrease of certain parameters, such as pressure, amplitude, acceleration, etc. with increasing distance from the center of vibration. Others have approached the problem from the use of radius of action of the vibrator, and still others have tried to relate the frequencies of vibration to the strength and density of the vibrated concrete. The results of these investigations are hereby reviewed.

Studies of Attenuation of Vibration

Studies of attenuation have involved measuring the changes in acceleration, amplitude, radius of action, pressure, etc. as the distance of a point from the vibrator increses. Several of the investigators used different types of transducers immersed in the concrete at selected locations to carry out their measurements (9), (10), (15). These studies showed that the attenuation of vibration is very rapid at points close to the vibrator and decreases very rapidly as the distance from the vibrator increases. See Figures 5 to 7. There appears to be a general agreement as to the form of attenuation of the above parameter. There is still need to determine which parameters affect the compaction of concrete and how they do so.

Many investigators have viewed acceleration as the most significant parameter in compacting concrete (16), (17), (18) with suitable values ranging from 4g to 7g[3]. However, Cussens and Plowman (19), (20) suggested that the importance of acceleration as observed in clamped molds of small masses might not be equally applicable to the larger and more steady masses of the type which must be vibrated on the field. Cussens found that velocity was a significant variable when frequency was kept constant, but Green (17), in rebuttal, said that this was true only because velocity is a function of both amplitude and acceleration. In fact, it appears that the researchers were simply using different terms to express the same conclusion. It

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[3] g is the acceleration due to gravity.

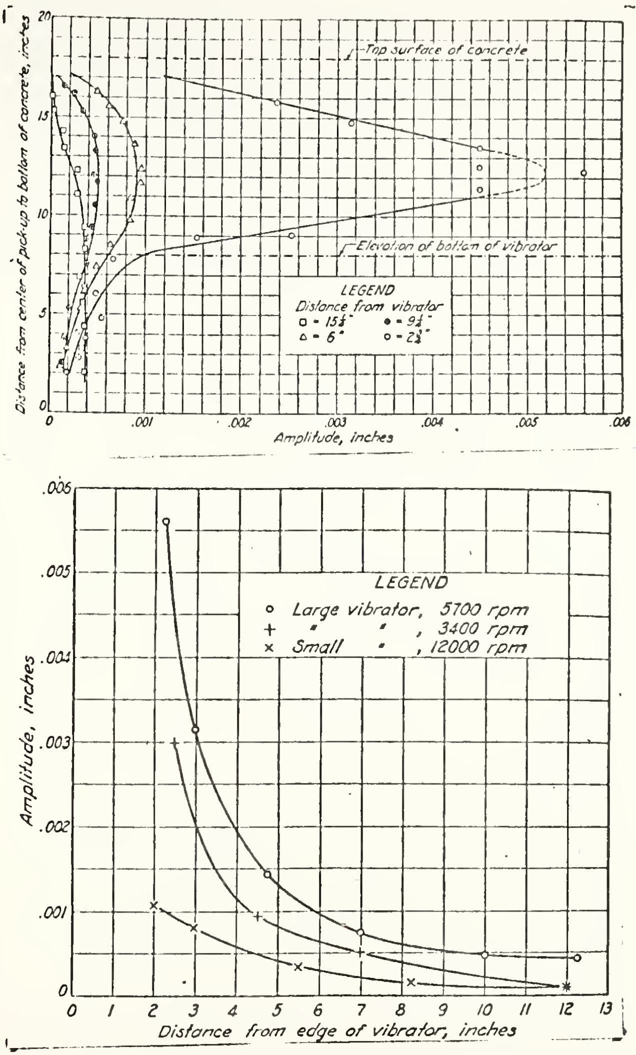


Figure 5

Pigman and Hornibrook's Decay of Amplitude Curves

(ref. 15)

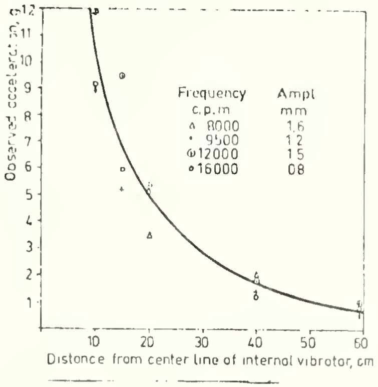


Figure 6

Las Forssblad's Decay of Acceleration Curve

(ref. 10)

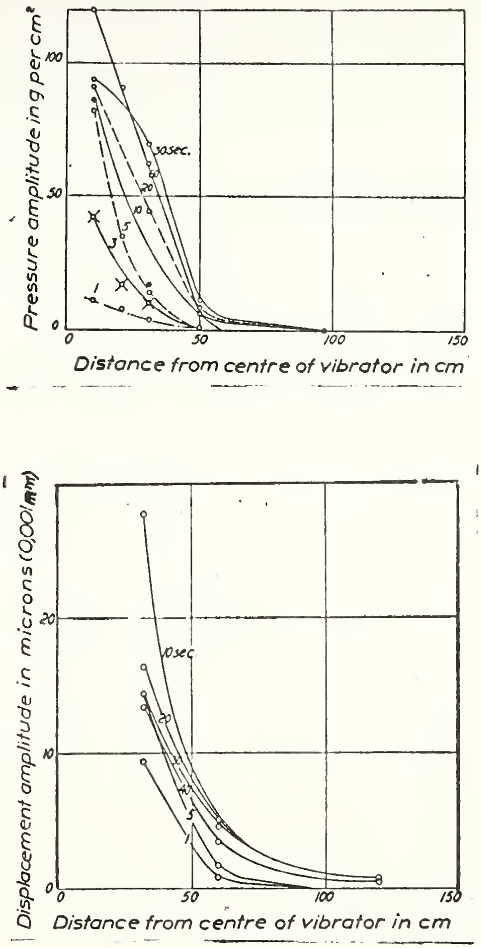


Figure 7

Bergstrom's Decay of Amplitudes of Pressure and Displacement Curves

(ref. 9)

is well known that acceleration is directly proportional to the square of the frequency of vibration and amplitude is determined by this constant divided by the acceleration (20). Therefore, a study of the effects of acceleration encompasses both frequency and amplitude of displacement of vibration.

Other investigators such as Bergstrom (9) and Las Forssblad (10), believed that increase of amplitude increased the radius of action of the vibrator and thus the effective volume of concrete compacted. Forssblad found that the radius of action increased with frequency up to 10,000 cpm and slowly increased above this value up to an optimum value of 12,000 cpm before starting to drop (see Figure 8). He found that an increase in amplitude caused an increase in the radius of action at every frequency. He also noted that those vibrators with the greatest radii of action had produced concretes which, in the hardened state, had the highest compressive strengths. On the other hand, Bergstrom could not detect any relationship between the radius of action and frequency, amplitude, or centrifugal force, if the quantities were considered separately. However, he noted that the energy consumed per unit volume of fully compacted concrete increased as the radius of action decreased. This observation supports the finding by Las Forssblad (10) and Saul (21), if it is noted that the radius of action increases with time of vibration (see Figures 3 and 9). Saul and Forssblad concluded that power input increased as the cube of the frequency and as the square of amplitude. It can be concluded that amplitude, or acceleration, has an influence on the compaction of concrete, which diminishes as the distance from the vibrator increases.

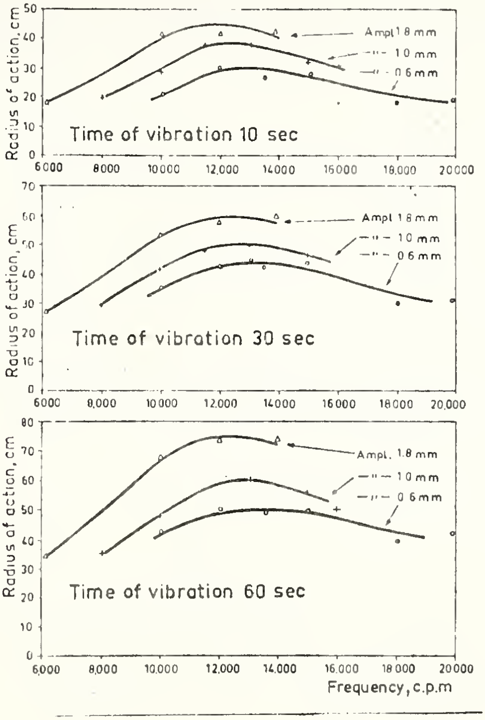


Figure 8

Las Forssblad's Curve for the Relationship Between Radius of Action, Frequency and Amplitude

(ref. 10)

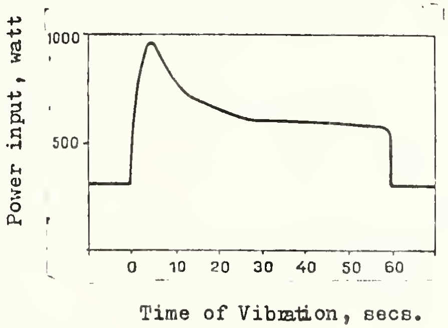


Figure 9

Power Input of an Internal Vibrator During a Single Immersion in Concrete

(ref. 21)

Studies of Frequency

Other investigations studied the frequency and attempted to relate amplitude and frequency to the amount of compaction. Unfortunately, little use can yet be made of the mass of data gathered so far.

Las Forssblad (10) has indicated the existence of an optimum frequency beyond which concrete compaction should be discontinued since it may result in the segregation of the mix. (See Figure 8 , where this frequency was 12,000 cpm). The existence of such an optimum frequency is also supported by studies of Nessim and Wadja (5) who identified "a critical rate of shear beyond . . . which only a small increase in the compaction of concrete is achieved ". Wadhwa (22) also obtained the same optimum frequency as Las Forssblad, and found that for every frequency there was a corresponding amplitude which gave the maximum degree of compaction. Green (17) and Cussens (19) further confirmed the existence of such an optimum frequency. All of these investigators agreed that the strongest concrete is produced at the optimum frequency. The latter two investigators used table vibration whereas the former investigators used internal vibrators. Bennett and Gokhale (16), even though they did not postulate the existence of an optimum frequency, found that a frequency at constant power input had a "remarkable" effect on the strength of cement mortar and paste.

Other investigations have been carried out to determine the best combinations of frequency and amplitude for internal vibrators. Their findings suggest that relatively low frequency, high amplitude

vibrators are suitable for dry concrete mixes in the first phase of compaction. Vibration with high frequency and low amplitude has been found to have the greatest effect during the second phase, presumably because it will cause more rapid expulsion of entrapped air (2). However, a study which measured the mechanical impedance of concrete concluded that frequency might not be a critical parameter in concrete vibration (12).

Studies Involving Surface Vibration

The experimental studies described so far were carried out with several sizes, shapes and types of molds which were attached to vibrating tables, and with various types and sizes of internal vibrators. However, as pointed out earlier, modern highway constuction equipment uses both internal and surface vibrators for concrete compaction. It is therefore desirable to review the published works on surface vibration.

There is consensus among the few investigators who have studied surface vibration that the main factors affecting the compaction of concrete are the amplitude of vibration, the number of vibrations transmitted to the concrete (i.e. the frequency of vibration divided by the forward speed of travel), and the force on the concrete. The maximum thickness of the slab which can be properly compacted is related to the product of these three factors (24), (25).

It was also found that the compaction varied with the depth and the consistency of the mix. The limiting value of the depth of compaction was found to be about 12 inches. The acceleration of the surface vibrator appeared to have no effect on the compaction (24),

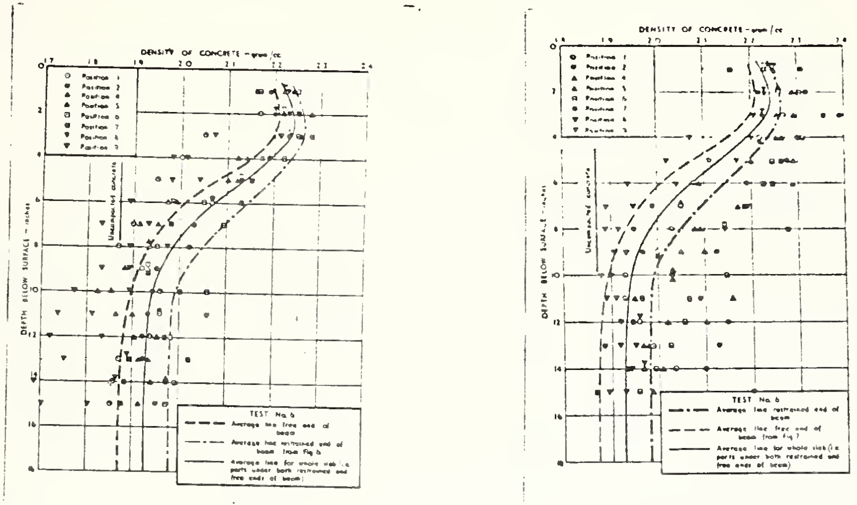
a finding which is contrary to the results obtained with vibrating molds and internal vibration. It was also found that there was a relationship between the density and the particle velocity, another relationship not noticeable with table or internal vibration. Figure 10 shows the variation of density with depth of compaction.

Studies Involving Reinforcement

The discussions so far have dealt with plain concrete. Laboratory work conducted so far has indicated that the presence of reinforcement decreases the radius of action of a vibrator (10). Studies of the surface vibration of reinforced concrete have led to the conclusion that the presence of reinforcement affects the depth of compaction of concrete (24). The concrete in a reinforced slab was found to be fully compacted down to the reinforcement, beyond which its density gradually dropped off. The density of the bottom of the slab was found to be about 35% of the density of the top one inch of thickness (26). (See Figure 11). These findings seem to indicate that laboratory experiments which exclude reinforcements cannot be immediately applied in the field where practically all road slabs presently built are reinforced.

Investigations in Various States

Several states have conducted research on various aspects of concrete vibration, and their findings have been reported to the coordinating committee of the National Experimental Evaluation Program. The results of the completed studies have been reported in



The variation of density with depth for nine sections in one slab under the free end of the vibrating beam.

The variations of density with depth for nine sections in one slab under the restrained end of the vibrating beam.

Figure 10

Variations of Density with Depth of Compaction

(ref. 24)

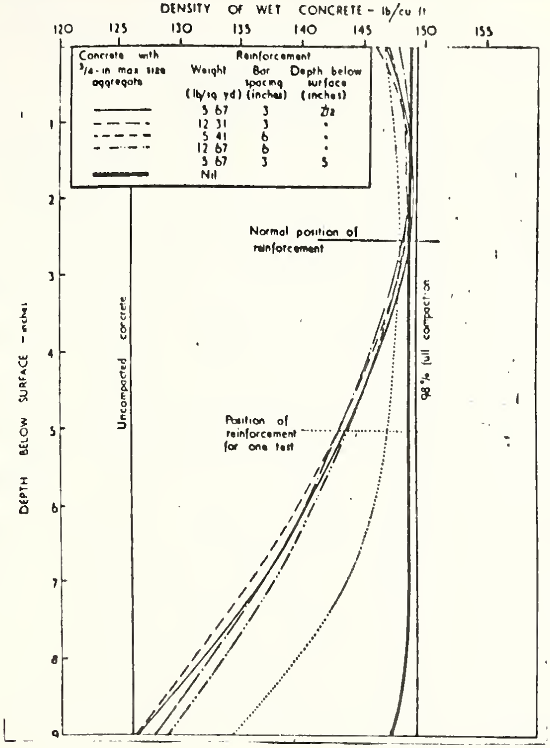


Figure 11

Effect of Reinforcement on Depth of Compaction of Concrete

(ref. 26)

various publications (27), (28), (29), (30). Work is still in progress in several states, including New York, Kentucky and Utah. Various conclusions have been reached and various recommendations made.

General Comments

The literature review suggests that, when concrete is vibrated, there is a "coupling" between the vibrator and the concrete whereby the vibrator transfers energy to the concrete. Some minimum level of initial energy appears to be necessary in order to initiate this energy transfer, since very low frequency and high amplitude do not compact concrete. Kolek (8) found that the initial frequency should be at least 6,000 cpm and the diameter of the head of an internal vibrator should be between "35 and 40 cm" (sic) (10). Texas studies suggested that frequency should be at least 7,000 cpm. The intial breakdown occurs within the first few seconds of vibration; continued vibration after this will fluidify the paste and cause the aggregates to move to optimum positions. Maximum compaction is achieved at an optimum frequency of 12,000 cpm (5), (10), (17), (19), (22), (23). Vibration at still higher frequencies results in less compaction and possibly leads to the segregation of the mix. Vibration at a frequency higher than 12,000 cpm substantially reduces the radius of compaction, since the concrete mass becomes fluid before it can "respond" to the rapid displacements which are imparted by such rapid vibration.

The studies up to the present have not been able to define frequency as a function of the density or the pulse velocity in the

mix. Some authors believe that there is a reduction in the frequency or acceleration when a vibrator is inserted in the mix as compared to when it is outside the mix. It has, however, been shown that the physical parameters of concrete, such as consistency, influence compaction more than the characteristics of the vibrator (8), (10), (27).

A major drawback of most of the research studies is their lack of rigorous statistical analysis, wherein the validity and the universality of application of their findings can be assessed. Secondly, most of the studies, except the most recent ones in the United States, have been directed towards solving general vibration problems rather than towards solving highway pavement construction problems. As a consequence, their findings have not been immediately usable to the field engineer who wants to know what frequency he should use during vibration, what the slump range should be for best vibration results, how he is to determine whether he is getting proper vibration, what range of density he should obtain in the field in plastic concrete so that he can be sure he is getting proper compaction below the steel, etc. The study conducted by the author has attempted to answer some of these questions. It has also attempted to simulate the field conditions surrounding highway pavement construction to the extent possible within the confines of a laboratory and reasonable costs.

Characterization of Concrete Material

Before going to the description of the experiment, the author feels it necessary to clarify the characterization of concrete. Several questions have been asked as to whether this material is a

liquid, slurry, or something else. The characterization herein proposed is that of a non-Newtonian fluid which is thixotropic.

A fluid is a substance that deforms continuously when subjected to a shear stess (31). Fluids may be either Newtonian or non-Newtonian. Newtonian fluids obey Newton's law of viscosity which states that the stress in a fluid due to an applied force is directly proportional to the rate of angular deformation in the fluid. Fluids that do not obey this linearity law are called non-Newtonian fluids. In one particular class of non-Newtonian fluids, known as the viscoelastic type, the rate of strain is dependent upon both the shear stress and the amount of strain. A non-Newtonian fluid whose strain rate is a function of both the stress magnitude and the stress duration is said to be thixotropic (32). The viscosity in a thixotropic fluid is thus dependent upon the immediately-prior angular deformation of the fluid. Such fluids can be agitated and used in a workable condition, but beoome immobile after the stress is removed. Figure 11 shows the relationship between the rate of deformation and shear stress for both the Newtonian and non-Newtonian fluids. Examples of Newtonian fluids include water, common oils, and gases, whereas non-Newtonian fluids include pigment suspensions, paint, printer's ink, and asphalt tar. The rate of deformation of tar is so low that the tar will sustain a load, such as a piece of aggregate, on its free surface.

At this juncture, the theoretical characterization of cement paste, within the water-cement ratio limits used in slipform paving, is readily apparent. It is a thixotropic substance with such a low

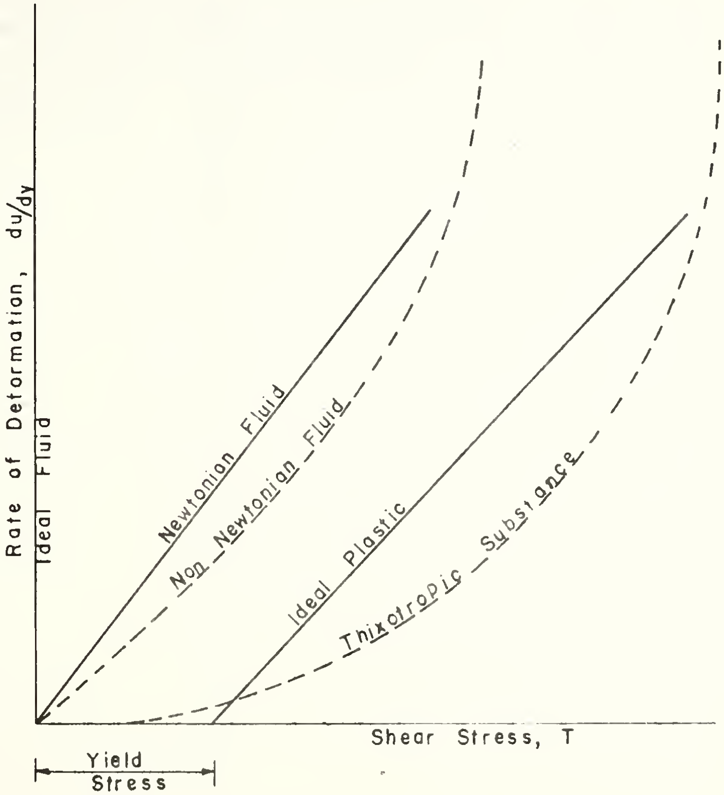


Figure 12

Relationship Between Shear Stress and Rate of Deformation for Newtonian and Non Newtonian Fluids

(ref. 31)

rate of deformation that it will support or sustain appreciable loads, such as aggregate particles embedded within the paste. It can be agitated, by mixing, into a workable condition but becomes immobile as soon as this source of stress is removed. This immobility due to the thixotropic action will persist until it is superseded by the rapid chemical process that follows. The experimental findings of Nessim and Wadja (6), as shown in Figure 13, support this theoretical characterization.

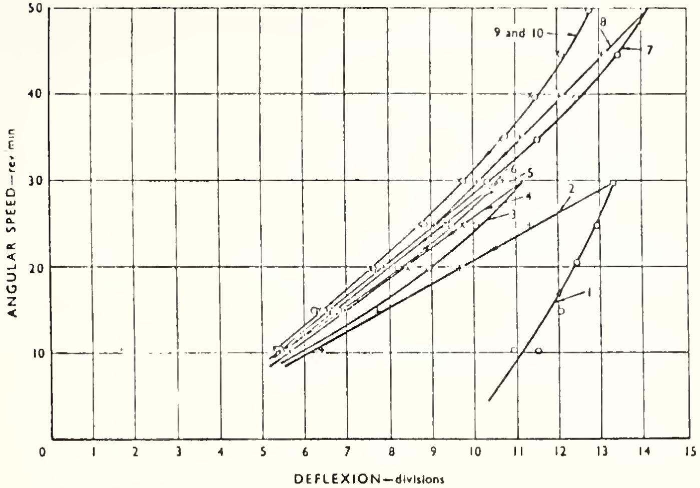


Figure 13

Nessim and Wadja's Rheological Curves for Cement Paste

(ref. 6)

DESIGN OF EXPERIMENT

As noted earlier, this research was conducted under conditions which simulate actual field conditions as closely as possible. It is hoped that it will be possible, with minimal modifications, to implement its findings on actual construction projects. However, it should be borne in mind that the research programs discussed in the literature and briefly reviewed herein have in total required several years of continuous experimentation. A research effort of the scope to be presented here can seek to answer only a few of the many questions about concrete vibration.

Plan of Experiment

This experiment was initially conceived to involve both a laboratory phase and a field investigation. However, funding was obtained for only the laboratory phase necessitating that the two phases be independent of each other. The preliminary planning for this phase contemplated the use of six different concrete mixes, namely, crushed limestone aggregate with 1) 38% sand[4], 2) 33% sand, 3) 38% sand plus water reducer (lignosulfonate), 4) 38% sand plus flyash replacing some of the cement, and gravel aggregate with

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[4] $$\text{Percent sand} = \frac{\text{Wt. of Sand}}{\text{Wt. of Sand + Wt. of Coarse Aggr.}} \times 100$$

5) 33% sand, and 6) 38% sand. Five different periods of vibration were specified, all within a range of 5 and 13 seconds which was considered representative of concrete compaction periods for slipform pavers in actual use. Five different frequencies of vibration, varying between 7,200 to 13,200 cpm, were also selected. This frequency range was chosen after studying the specifications of several states, (see Figure 2), and the capabilities of the available vibrators and generator. From the literature review, it was decided that the measurements of density and pulse velocity would be the acceptable means for verifying the effectiveness of various vibratory frequencies. The concrete parameters which were to be measured consisted of the slump, air content, compaction factor, Vebe time, and temperature of concrete at placement, with cylinders cast for strength tests. Particle acceleration was to be measured by embedding accelerometers in the mix, the frequency of the vibrator was to be measured both in air and in concrete, and the amplitude of the vibrator was to be measured by attaching an accelerometer to it.

Consistent with this initial planning for the laboratory phase, three main variables to be investigated were mix (6 levels), frequency (5 levels), and duration (5 levels). It was realized that a complete factorial experiment would involve 150 test runs, even without the replication which was considered essential if the laboratory results were to be applied to field operations. Each replication would, of course, require an additional 150 test runs. It was then decided to consult with a statistician.

A completely randomized factorial design was quickly ruled out because it would require a very large number of test runs. Instead,

the findings of Akroyd (2), Forssblad (10), and Bower and Gerhardt (27) that the range of frequency for best compaction was between 10,800 and 12,000 cpm were used to develop a composite design for the experimental work.

Composite Statistical Design for Experimental Work

A composite statistical design for k factors has three parts: 2^k points representing a basic two-level factorial or fractional factorial, a single point at the center of the entire design, and 2k points which are successively located in pairs at the two extremes of each of k factors while remaining centered for all other factors. For this type of design, therefore, there are $2^k + 2k + 1$ treatment combinations (33). The two-level factorial (2^k) provides information on the main effects (linear) and the interactions (linear x linear) around the center. The end points (2k) provide information for the quadratic (or squared terms) fit across the full span of the factor levels. The center point is the estimated optimum combination for all factors in the experiment. Two factors, frequency and duration, were investigated for each of the mix designs used in the experiment. The "center point" (or assumed optimum) values for these two factors, in combination, were taken as 10,800 cpm at 9 seconds duration. Provision was also made for two extra experimental units for the center point. The composite statistical design for a single series of tests on each mix design thus involved eleven experimental runs ($2^2 + 2x2 + 3 = 11$). For the six mixes which were to be investigated, still assuming a single replication of tests, this would mean a total of 66 runs as compared with 150 runs required for a complete

factorial experiment.

It was initially intended to have three replicates of all test runs, thereby obtaining 33 values which could be used in determining the regression equation for each mix. However, because of physical limitations in the laboratory, the experiment could be run with only one mix, one frequency, one duration and two replicates at a time. This necessitated the use of a split-plot design (33) wherein the whole plot was the replication (R), the split was the mix (M) and the split-split plot was the frequency (F). Each of these levels or restrictions on randomization constitutes an experimental block.

Finally, to fix the levels of the factors in the composite design, a rotatable design (33) centered at the estimated optimum center point was adopted. The factorial points moved away from this center in both directions and the end points were located at the ends of the factor spaces. The arrangement is shown diagramatically in Table 1 and Figure 14.

After the details of the composite design had been determined, the next step was to randomize the experimental units within each mix and for each of the three replications. In this context, an experimental unit is viewed as the material which is to receive the various experimental treatments (33). In this experiment, the experimental units were the reinforced concrete specimens which were to be cast, vibrated and tested. The restrictions on randomization which were imposed by the physical laboratory limitations made it necessary to protect the experimental units from any bias which might be introduced by some unknown factor. This need was satisfied by randomizing the experimental units within the constraints introduced

Table 1: Treatment Combinations Used in Composite Statistical Design

Composite Factor	Duration (secs.)	Frequency (cpm)	Number Used in Randomization
Center Point	9	10800	1
Factorial	7	9600	2
	7	12000	3
	11	9600	4
	11	12000	5
End Points	5	10800	6
	13	10800	7
	9	7200	8
	9	13200	9

by this space restriction. After randomization in this manner, it was felt that any major bias would be eliminated from the estimates of such parameters as nuclear density, pulse velocity, etc., and from the tests for significance of the effects of the treatments on the variable being analyzed.

To carry out the randomization, the numbered mix designs described on page 33 were randomly allocated to random numbers from 1 to 6. The results of this randomization are shown in Table 2.

Table 2: Randomization of Mix Designs

Random Number	1	2	3	4	5	6
Mix Number	5	6	3	1	4	2

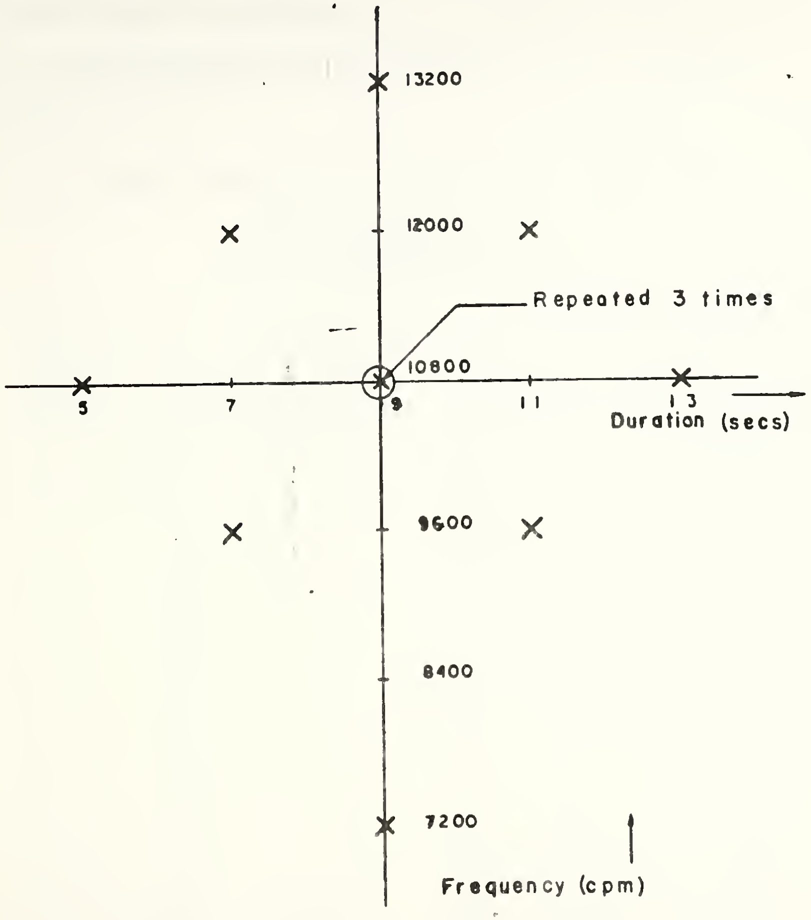


Figure 14

Factor Space Coverage of Experimental Variables

The treatment combinations shown in Table 1 were then assigned random numbers between 1 and 9, and a table of randomly assigned testing sequence was generated for each replication within each mix. The testing sequences which were generated for Mix 1 are shown in Table 3.

Table 3: Order of Carrying Out Tests on Experimental Units

Mix Number 1
(Crushed stone with 38% sand)

Replication No. 1			Replication No. 2			Replication No. 3		
Testing Sequence	Treatment Combination		Testing Sequence	Treatment Combination		Testing Sequence	Treatment Combination	
1	Freq.	Dur.	2	Freq.	Dur.	3	Freq.	Dur.
3	12000	7	5	12000	11	3	12000	7
9	13200	9	4	9600	11	9	13200	9
6	10800	5	9	13200	9	4	9600	11
2	9600	7	7	10800	13	2	9600	7
7	10800	13	3	12000	7	7	10800	13
4	9600	11	8	7200	9	5	12000	11
5	12000	11	1	10800	9	8	7200	9
8	7200	9	6	10800	5	1	10800	9
1	10800	9	2	9600	7	6	10800	5

After this randomization was completed for each mix and replication, a schedule of testing program was set up and each experimental unit was coded on the basis of the frequency, duration, replication number, mix number, and whether the unit had accelerometer embedded or not. The scheduling also included proposed dates of performance

and the actual date of performance.

Subsequent Modification to Composite Statistical Design

The random sequencing of mix designs, as shown in Table 2, had identified Mix 5 as the starting point for the experimental work. This was not feasible because crushed stone was already stored in the laboratory and additional space was not available to store the gravel required for Mix 5. Therefore, the sequencing of the experimental work was revised on the basis of a new randomization involving only mixes 1 to 4 inclusive (mixes utilizing crushed stone as coarse aggregate).

Additional modifications to the composite design were introduced while the experimental work was underway. Three mixes (crushed stone with 33% sand, crushed stone with 38% sand plus flyash, and gravel with 33% sand) were eliminated when unforeseen circumstances made it impossible to include these test series within the budgeted dollars and time. For the same reason, the number of replications was reduced from 3 to 2.

The concrete batches for the experimental units were initially produced at slumps between 1 and 2 inches. However, after the laboratory work was underway, it was discovered that the nuclear density and the pulse velocity readings were not varying appreciably as the level of frequency was changed. For this reason, concrete with a slump of 0 to 1 inches was used on most of the test runs for Mix 6 (38% sand with gravel coarse aggregate).

APPARATUS AND EQUIPMENT

General Apparatus

Descriptions of the slump test procedures (ASTM C 143-66), the Vebe equipment, and the compaction factor apparatus already appear in the literature and will not be repeated here (34), (35). Instead, this discussion will be confined to the forms, the vibrators, the generator, the nuclear gauge and the V-scope equipment.

Forms

The type of forms to be used in the experiment was determined after considering several factors. It was desired that a minimum vibration energy be transferred or reflected from the forms, that the specimen size permit a vibrator spacing such as is commonly used on slipform pavers, and that the forms be sufficiently rugged to allow 300 or more specimens to be cast.

To check the effective radius of action for the vibrator, three sizes of cylindrical fiber board forms were investigated. They were 16, 18, and 20 inches in diameter and provided for a specimen 8 inches in height plus a surcharge of 4 inches of concrete. The surcharge was added to simulate the field condition wherein slipform pavers carry a surcharge of concrete just prior to vibration. The fiber board forms were rejected for use in the experiment because they were not reusable and were not adaptable for simulating slipform paving

operation because of their cylindrical shapes. A set of 18" x 20" wooden forms, 12" high (8" for pavement and 4" for surcharge) was then investigated. This was also rejected because it was not considered rugged enough for the total number of tests to be run.

The form finally adopted was the same size as the wooden form, but was made of steel. It was designed so that, after vibration, the surcharge part could be lowered and the excess concrete screeded off to leave an 8 inch thick specimen. The form was supported on a base material meeting the requirements of Indiana State Highway Specification for No. 53 base material. The base material was compacted by the use of an air hammer and was prepared as close to field specifications as possible. The form was equipped with a railing to support the vibrator bracket and to facilitate the movement of the vibrator into and out of the concrete. Seven sets of forms were made; Figure 15 shows the details of a typical set.

Vibrators

The operation, types and fundamentals of vibrators have been described in the literature, and therefore will not be reviewed here (36), (37). The two vibrators used on this project were Model DHC 100B high cycle, motor-in-the-head types and were donated by the Dart Division of the Koehring Company. Each vibrator had a rotary-centrifugal eccentric rotating on high speed bearings. The eccentric was $1\frac{7}{8}$ inches in diameter and weighed 1.365 lb. The unit operated on 3 phase, 180 cycle current (as compared with regular single phase household current of 60 cycles) and could be used with either 115 or 230 volt generators. The head was $2\frac{1}{2}$" in diameter, $21\frac{1}{2}$" in length and weighed 20.375 lb.

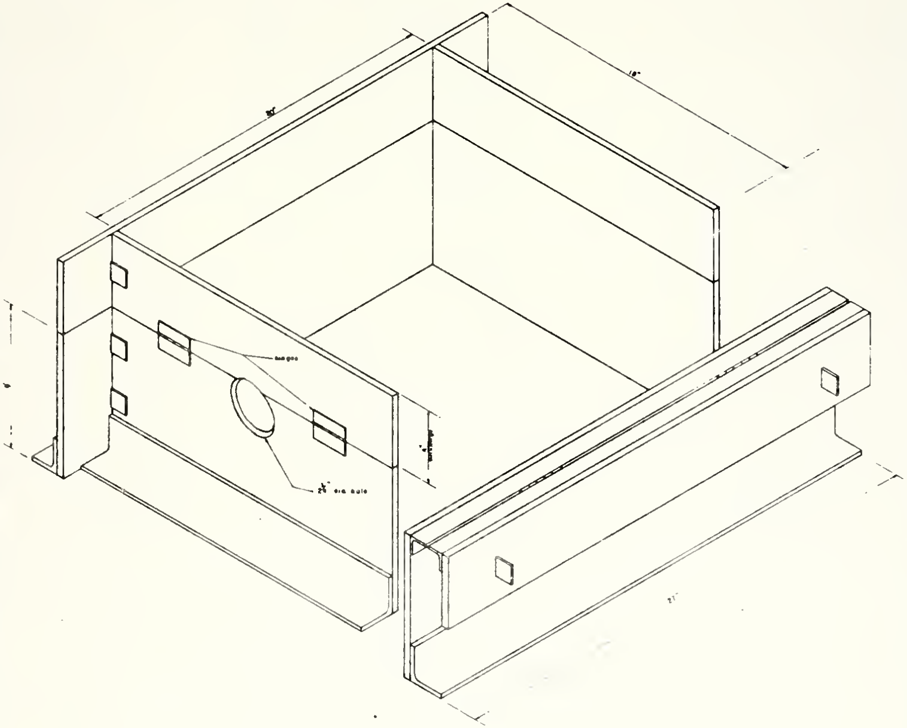


Figure 15

Details of the Forms Used

Figure 16 shows the details of the vibrator and its associated parts.

Generator

The generator used on this project was a Kohler Model 181 gasoline, single cylinder, 4 cycle air-cooled power plant. It was also donated by the Koehring Company. The unit developed 8 hp at 3600 cpm and operated on 180 cycle 3 phase alternating current. Its features included a solid ring rotating armature, saturated pole construction which assured good voltage regulation, and an adjustable governor for the regulation of frequency to within 5%.

V - Scope Apparatus

The pulse velocity measuring equipment was a Model C-4960 V - Scope manufactured by James Electronics of Chicago, Illinois. It consists of a cathode ray tube and two transducers which house some piezoelectric crystals. The procedures followed during use in this study conformed to ASTM C-597-71. Figure 17 shows the basic operation details of the V - Scope.

Nuclear Density Gauge

Nuclear gauges have long been used for density and moisture measurements in soils. They are now used for similar measurements in asphalt, soils, asphalt-concrete and more recently in concrete. A Nuclear Chicago gauge, borrowed from the Indiana State Highway Research Laboratory, was used in this investigation. The gauge consists of two parts, a Model 5901 probe and a Model 5200 scaler, and can

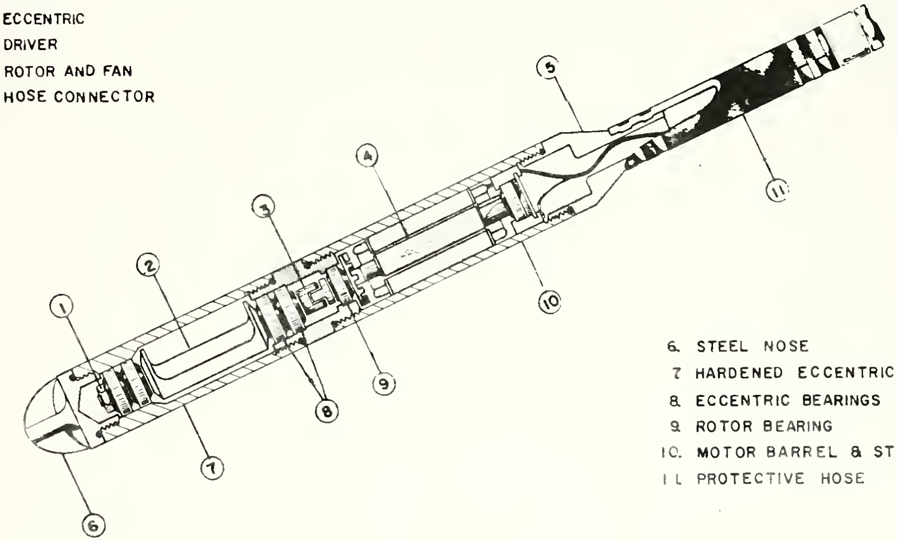
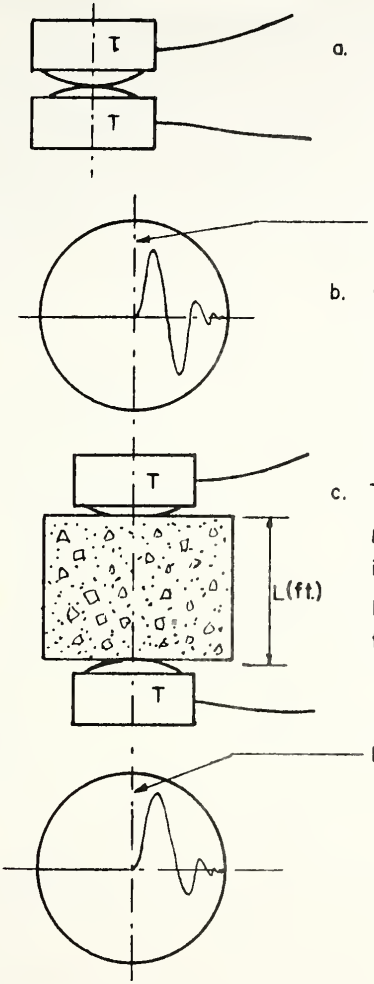


Figure 16

Details of the Vibrator and Its Associated Parts

(Dart Corp)



a. Instrument delay time is measured with both transducers (T) face to face with couplant

b. Counter is zeroed where waveform leaves the horizontal axis

c. Transducers are placed across the predetermined length (L), the waveform is again adjusted to point where it leaves the horizontal axis, then the time t, microseconds, is read

$$V = L/t$$

V = pulse velocity in ft/sec.
t = time through specimen, msecs.

Figure 17

Basic Operation Details of the V-Scope

(James Electronics)

perform both moisture and density measurements. Only the density measurements are of interest here.

The probe unit contains a four millicurie radium-beryllium source of radiation. This source is doubly encapsulated and emits both gamma and neutron radiation. The detectors are two halogen-quenched Geiger detector tubes connected in parallel. The shielding is a combination of lead and tungsten, and completely surrounds the source. The source and the shield are housed in a heavy-gauge metal case.

In conjunction with the with the probe and the scaler, a standard granite block of known density is provided for standardizing the gauge at the start of each operation. Figure 18 shows the probe with a 20 ft cable.

The gauge utilizes the principle of gamma rays back-scattering for density measurement. According to this principle, only a percentage of the gamma rays emitted by the radioactive source is reflected back to the Geiger detector tubes and counted. This percentage decreases as the density being measured increases (38).

The gauge was calibrated at the factory on six specimens of known densities and on the granite block supplied with the instrument. The Indiana State Highway Research and Training Center, prior to loaning it to this project, again checked its calibration on six specimens of known densities and the curves developed from this check were compared with the factory-supplied curves to assure the accuracy of the gauge's performance. Similar checks were made periodically throughout the period when the gauge was in use in the laboratory and at the completion of all laboratory tests. The average of the counts

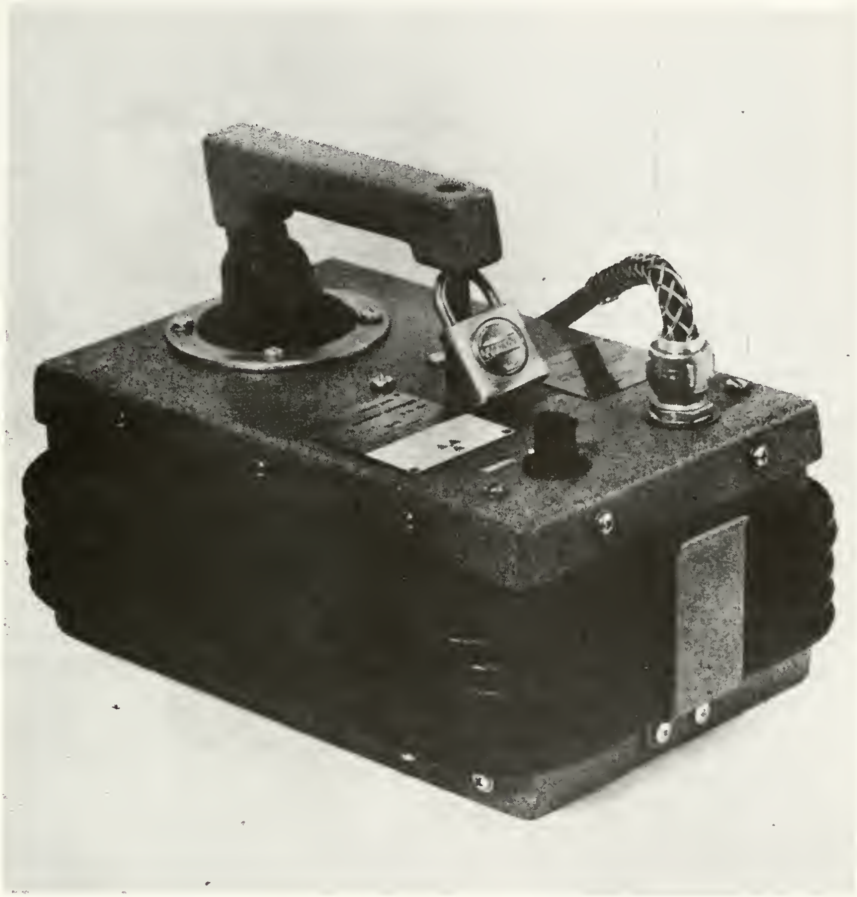


Figure 18

Nuclear Chicago's Density Probe

(Nuclear Chicago Corp)

on each of the blocks for all the checks made was used in developing a final calibration curve from which the densities of each of the specimens made during the experiment were calculated.

Calibration was carried out by taking ten one-minute density standard counts on the granite block. The standard counts were means of comparing the gauge's response to a predetermined response, in this case the granite block. Then the gauge was placed on one of the six standard blocks selected at random. Four successive one-minute counts were taken on each block, turning the gauge 90 degrees clockwise after each one-minute count. This procedure was repeated for each of the six specimens. Another density standard count was then taken and the average of the two was obtained. The average of the four counts on each specimen was then obtained and the count ratio, the ratio of the average of one-minute readings to the mean standard count, was obtained for each specimen. This ratio was taken to the calibration curve and the density of the specimen was read off. The calibration equation was:

$$D = -78.363786\ X + 247.956611 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where:

D = Desired density

X = Count ratio

Figure 19 shows a typical data sheet for the gauge.

Gauge Model: 5901

Scaler Model: 5920

Gauge Serial No.: 249

Scaler Serial No.: 301

Date: 3 - 3 - 72

By: Olateju

Density Std. Count: 4703.0 (before) 4724.6 (after)

Mean Std Count: 4713.8

BACKSCATTER DENSITY				
Block Density in lbs. p.c.f.	One Minute Readings	Average One Min. Readings	Count Ratio*	Remarks
84.9	1: 9596 2: 9550 3: 9878 4: 9886	9752.5	2.07369	85.4
92.5	1: 9358 2: 9362 3: 9402 4: 9153	9318.7	1.97689	92.9
114.0	1: 7843 2: 7886 3: 8120 4: 7967	7954.0	1.68738	115.2
136.7	1: 6779 2: 6789 3: 6871 4: 6919	6839.5	1.45095	136.1
151.0	1: 5762 2: 5707 3: 5798 4: 5750	5754.2	1.22071	152.0
163.4	1: 5015 2: 5187 3: 5071 4: 5088	5090.2	1.07986	163.3

*Count Ratio = Average one minute reading divided by Mean Std. Count.

Figure 19

Typical Calibration Data Sheet for Nuclear Gauges

Instrumentation

Acceleration, frequency and amplitude have been identified by earlier investigators as three major factors which may affect the vibratory compaction of concrete (8), (9), (10), (15), (19), (20). Bergstrom and Davies concluded that acceleration might be the determining factor in the compaction of concrete. Their tests used molds attached to vibrating elements with upward and downward movements. Plowman (20), on the other hand, showed that acceleration in molds varied from the edges to the center. Sullivan(12) suggested that frequency might not be as important a determining factor in concrete compaction as was commonly assumed.

Reliable measurement of these factors in concrete requires that the measuring devices impart a minimum disturbance to the concrete, that they be smaller than the largest particle in the mix and that they have densities and surface characteristics similar to those of the aggregate particles so that they can move in the mix freely and in the same manner as the aggregate particles. As was mentioned in the literature review, the motions experienced by aggregate particles during vibration may be rectilinear, translational or rotational. It was therefore desired during this experiment that the measuring devices be able to measure each of these movements without appreciable "wandering" or rotation in the mix.

Since transducers are usually unidirectional, rotational movements can only be measured by three such devices mounted along each of the three cardinal axes. The costs involved in the use of large numbers of accelerometers and the necessary equipment to monitor their movements adequately were in excess of the available budget. As a

consequence, only three accelerometers were used. They were embedded in the mix at a height of one inch above the base of the specimen being tested, at three positions in the form and directly below the steel. Figures 20 and 21 show the positions where accelerometers were embedded in the mix. The response of each accelerometer was monitored on an oscillographic recorder with built-in signal amplifiers and conditioners. The displacement (amplitude, as many investigators call it) in the mix was measured by the use of a linear variable differential transformer (LVDT). The output of each of these measurements was digitized on a large area record reader-vertical (LARR-V) digitizer.

Accelerometers

The accelerometers used in this experiment were the Kistler's piezotron low impedance, top connected transducer Model 815-A2. Each is a compression type piezoelectric sensor which contains a seismic mass and a stack of parallel connected quartz crystal plates, mechanically isolated and assembled under pressure in a separate internal preload sleeve.

The acceleration to be measured is sensed along the symmetrical housing axis perpendicular to the mounting base. Depending upon the acceleration orientation, the seismic mass's inertia produces either a positive or negative compression force upon the sensor crystals, and the crystals produce a net charge output proportional to this force. Figure 22 shows the dimensions and the typical design of the accelerometers. Each accelerometer is hermetically sealed at the factory, but to prevent moisture from getting into the connection

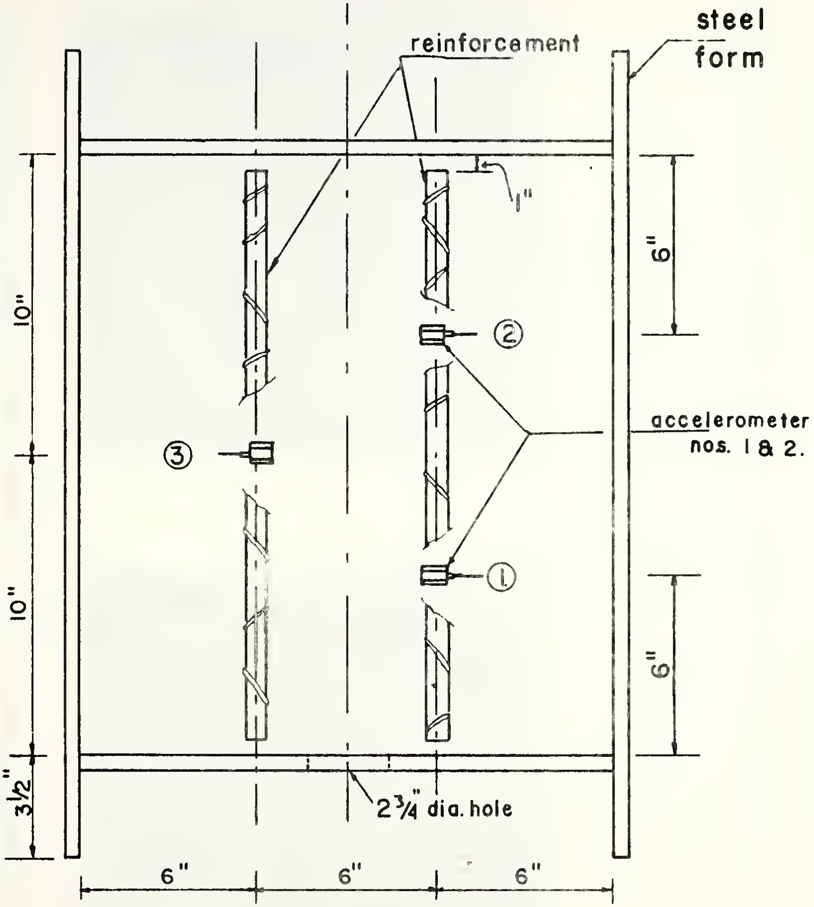
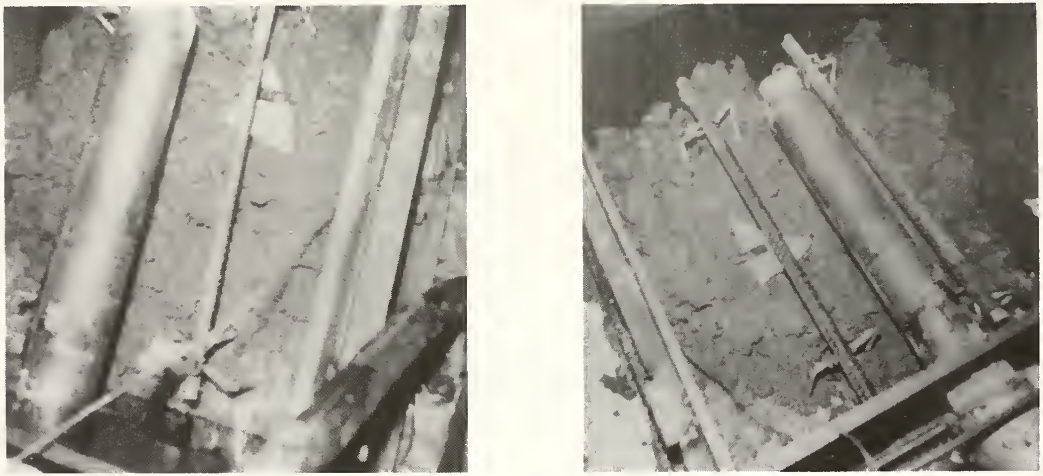


Figure 20

Schematic of the Positions of the Three Accelerometers in the Form



(position 2)

(position 3)

Figure 21

Locations of Accelerometers Relative to the Vibrator within the Form

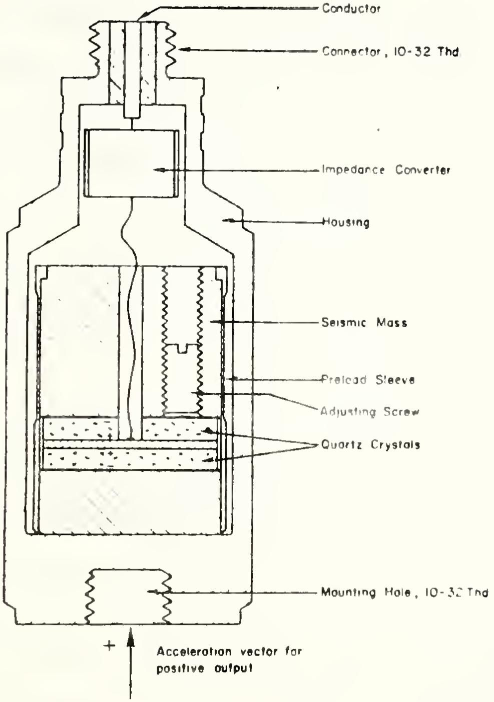


Figure 22

Schematic of Accelerometers

(Kistler Corp)

between the accelerometers and the coaxial cables, all the connections to the accelerometers were sealed in heat-shrinkage rubber tubing.

Oscillographic Recorder

The recorder used was a Dixon Southern Model 10-300 with 12 input or data recording channels. It is a direct recording system which operates by converting each electrical input signal from the accelerometer into an optical one. The fluctuations in the electrical signals are recorded as a series of traces which are produced by narrow beams of light from super pressure mercury lamps moving over the surface of a photosensitive recording paper as the paper is mechanically driven out of the recorder.

Each electrical input signal is fed to the coil of its own miniature galvanometer and all the galvanometers are mounted in a strong magnetic field. Variations in the amplitude of the electrical current through the coil alter the angle of deflection of the galvanometer mirror, so that the narrow beam of ultra violet light reflected by it appears as a spot of light and moves across the recording paper. The sideways movement of the spot is directly related to the amplitude of the signal current and the speed at which the paper is driven out of the magazine.

The recorder is equipped with facilities to draw grid lines parallel to the edges of the paper and vertical lines at preset time intervals. It also has traces and event markers for identifying the traces and marking specific occurrences during recording.

Other features include built-in amplifiers and signal conditioners, record duration and trace intensity controls. A paper take-up unit was purchased along with the recorder.

Linear Variable Differential Transformer (LVDT)

To measure the displacement amplitude of the vibrator at the tip and along the stem, an LVDT was used. This equipment was selected after other attempts to measure the displacement by accelerometers failed. This aspect is discussed in another portion of the report.

The LVDT is an electro-mechanical transducer which produces an electrical voltage output proportional to the displacement of a separate movable core. It consists essentially of three equally spaced coils on a cylindrical coil form. A rod-shaped magnetic core positioned axially inside this coil assembly provides a path for magnetic flux linking the coils. When the primary or central coil is energized with alternating current, voltages are induced in the two outer coils. The outer or secondary coils are usually connected in series opposition so that the voltages in a secondary circuit are opposite in phase, the net output of the transformer being the difference of these voltages. At the center of the core the output voltage will be zero. This is the null position or balance point.

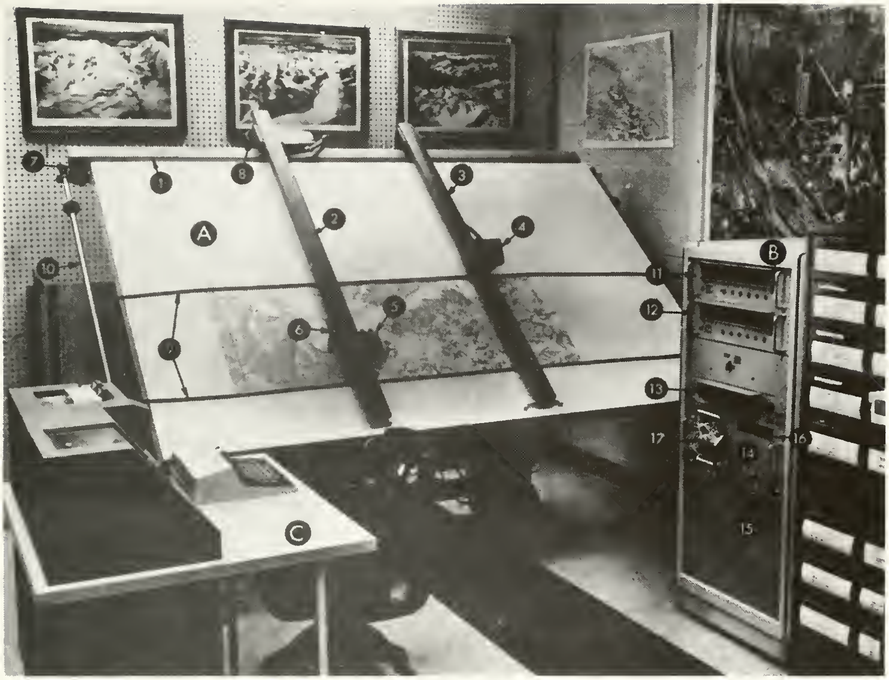
When the core is moved from the null position the voltage induced in the secondary coil toward which the core is moved increases, while the voltage induced in the opposite coil decreases. The differential voltage thus produced varies linearly with the change in core position. Motion of the core in the opposite direction beyond the null position produces a similar linear voltage characteristic, but with the phase shifted 180 degrees. There are several types, sizes,

ranges and physical configurations of LVDT's commercially available. In laboratory use, the LVDT was connected to the oscillographic recorder, and the records such as the accelerometer outputs were taken.

LARR-V Digital Coordinatograph

The data collected on displacement and acceleration were digtized for Fourier analysis on the Large Area Rapid Reader-Vertical (LARR-V) available in the School of Civil Engineering. It was felt that Fourier analysis provided a more accurate treatment of vibration data than merely measuring the peaks as suggested by Plowman (21, p 201).

The LARR-V is a digital coordinatograph which is capable of measuring distances along two mutually perpendicular axes, X and Y, and punching these values, along with accessory information, onto data cards. The equipment will accurately, rapidly and conveniently reduce graphical information to numerical data suitable for computer processing (39). Figure 23 shows the details of the LARR-V digitizer.



(ref. 39)

LEGEND

A THE READING TABLE
1 X-axis arm
2 Y-axis arm
3 keyboard arm
4 keypack
5 cursor carriage
6 readout button
7 X-axis encoder
8 Y-axis encoder
9 magnetic hold-down strips
10 roller for long charts

B TELECORDEX UNIT
11 X-axis telepack
12 Y-axis telepack
13 event counter
14 memory and control unit
15 power switches
16 constant data switches
17 patch panel

C IBM MODEL 029 CARD PUNCH

Figure 23

The LARR-V Digital Coordinatograph

DATA COLLECTION AND PROCESSING

When this experiment was in the planning stage, it was proposed that selected concrete and vibrator parameters be measured. These parameters include the slump, air content, Vebe time, unit weight, compaction factor, compressive strength, temperature of the concrete, acceleration, frequency, and amplitude of vibration. It was also proposed that the efficacy of each treatment combination when applied to each experimental unit be evaluated in terms of the nuclear density, pulse velocity, particle acceleration and displacement amplitude of the vibrator in the mix. However, before commencing the data gathering phase, some preliminary tests were necessary. These tests included evaluation of such physical characteristics of the aggregates and cement as specific gravity, moisture content and particle size distribution; design of the concrete mixes; calibration of various pieces of equipment; and positioning of accelerometers in the mix. The tests carried out on the aggregates were routine ASTM Standard tests, the description of which are available in the literature (40). However, the decision about the positioning and suspension of accelerometers involved a series of experiments where the supporting devices for the accelerometers varied from a cotton thread to a "massless" spring. None of these supports proved satisfactory because each introduced a new set of variables into the already complicated set of parameters which had to be dealt with in

the analysis. In addition, each support interfered with the free movement of the accelerometers and the aggregates whose motion it was simulating. Tests were then carried out to check the drift, lateral and vertical movements which might occur if the accelerometers were not restrained in any way. Since the observed movements were negligible, all of the tests subsequently carried out had the accelerometers positioned in the mix at the positions indicated in Figure 20 without any physical restraints.

During the preliminary tests, sieve analyses were performed on the aggregates. These were continued throughout the testing program as new deliveries of aggregates were received. Table 4 shows the gradation of the aggregates for the different deliveries and Appendix A shows the grading curves of the aggregates.

The concrete mix designs were also proportioned and tested in the laboratory. These designs were modified from time to time during the experimental runs to accommodate the variations in the physical conditions of the aggregates as they were received. Table 5 shows the general basic ingredients involved in each of the three mix designs which were ultimately used. The three mix designs, as mentioned previously, were Mix 1 which was crushed limestone with 38% sand sand, Mix 4 which was crushed limestone with 38% sand and a polymer-type water reducing agent, and Mix 6 which was a gravel mix with 38%sand. All the tests and mix designs were carried out in accordance with ASTM standards.

Table 4: Gradation of Each Delivery of Coarse and Fine Aggregates

Coarse Aggregates - Crushed Limestone

Delivery No. 1

Sieve Size	Weight Retained (lb)	Cummulative Wt. Retained (lb)	Percent Retained (%)	Cum. Pct. Retained (%)	Percent Finer (%)
1½"	0.30	0.30	0.1954	0.1954	99.8046
1"	25.90	26.20	16.8697	17.0651	82.9349
¾"	30.65	56.85	19.9635	37.0286	62.9714
½"	57.50	114.35	37.4519	74.4806	25.5194
⅜"	30.42	144.77	19.8130	94.2943	5.7057
#4	6.67	151.44	4.3444	98.6387	1.3613
< #4	2.09	153.53	1.3613	100.0000	0.0000
Delivery No. 2					
1½"	0.00	0.00	0.0000	0.0000	100.0000
1"	3.28	3.28	7.8860	7.8860	92.1140
¾"	6.59	9.87	15.8450	23.7320	76.2680
½"	15.94	25.81	38.3270	62.0580	37.9420
⅜"	9.80	35.62	23.5630	85.6220	14.3780
#4	3.91	39.52	9.4010	95.0230	4.9770
< #4	2.07	41.59	4.9770	100.0000	0.0000
Delivery No. 3					
1½"	0.00	0.00	0.0000	0.0000	100.0000
1"	4.34	4.34	13.2440	13.2440	86.7560
¾"	6.36	10.70	19.4080	32.6520	67.3480
½"	12.23	22.93	37.3210	69.9730	30.0270
⅜"	7.12	30.05	21.7270	91.7000	8.3000
#4	2.14	32.19	6.5300	98.2300	1.7700
< #4	0.58	32.77	1.7700	100.0000	0.0000

Table 4, cont.

Delivery No. 4

1½"	0.00	0.00	0.0000	0.0000	100.0000
1"	13.75	13.75	27.2650	27.2650	72.7340
¾"	14.58	28.33	28.9111	56.1771	43.8230
½"	15.22	43.55	30.1800	86.3570	13.6430
⅜"	4.92	48.47	9.7560	96.1130	3.8870
#4	0.97	49.44	1.9230	98.0370	1.9630
<#4	0.99	50.43	1.9630	100.0000	0.0000

Delivery No. 5

1½"	0.19	0.19	0.3950	0.3950	99.6050
1"	6.90	7.04	14.3570	14.7520	85.2480
¾"	8.34	15.43	17.3530	32.1057	67.8943
½"	15.78	31.21	32.8340	64.9400	35.0600
⅜"	9.55	40.76	19.8710	84.8110	15.1890
#4	3.83	44.59	7.9690	92.7800	7.2200
<#4	3.47	48.06	7.2200	100.0000	0.0000

Coarse Aggregate – Gravel

Delivery No. 1

1½"	0.00	0.00	0.0000	0.0000	100.0000
1"	6.97	6.97	13.9400	13.9400	86.0600
¾"	19.29	26.26	38.5800	52.5200	47.4800
½"	17.86	44.12	35.7200	88.2400	11.7600
⅜"	3.79	47.91	7.5800	95.8200	4.1800
#4	1.35	49.26	2.7000	98.5200	1.4800
<#4	0.74	50.00	1.4800	100.0000	0.0000

Table 4, cont.

Fine Aggregates

Delivery No. 1

Sieve Size	Weight Retained (gm)	Cummulative Wt. Retained (gm)	Percent Retained (%)	Cum. Pct. Retained (%)	Percent Finer (%)
# 4	7.50	7.50	0.3710	0.3710	99.6290
# 8	310.50	318.00	15.3670	15.7390	84.2610
# 16	436.50	754.50	21.6040	37.3420	62.6580
# 30	492.50	1247.00	24.3750	61.7170	38.2830
# 50	574.00	1821.00	28.4090	90.1260	9.8740
#100	161.50	1982.50	7.9930	98.1190	1.8810
<#100	38.00	2020.50	1.8810	100.0000	0.0000

Fineness Modulus = 3.034

Delivery No. 2

# 4	3.00	3.00	0.3000	0.3000	99.7000
# 8	115.00	118.00	11.5000	11.8000	88.2000
# 16	203.00	321.00	20.3000	32.1000	67.9000
# 30	261.00	582.00	26.1000	58.2000	41.8000
# 50	306.00	888.00	30.6000	88.8000	11.2000
#100	92.00	980.00	9.2000	98.0000	2.0000
<#100	20.00	1000.00	2.0000	100.0000	0.0000

Fineness Modulus = 3.108

Delivery No. 3

# 4	4.00	4.00	0.4000	0.4000	99.6000
# 8	104.00	108.00	10.4000	10.8000	89.2000
# 16	161.00	269.00	16.1000	26.9000	73.1000
# 30	258.00	527.00	25.8000	52.7000	47.3000
# 50	350.00	877.00	35.0000	87.7000	12.3000

Table 4, cont.

#100	96.00	973.00	9.6000	97.3000	2.7000
∠#100	27.00	1000.00	2.7000	100.0000	0.0000

Fineness Modulus = 3.242

Delivery No. 4

# 4	5.00	5.00	0.5005	0.5005	99.4995
# 8	118.00	123.00	11.8118	12.3210	87.6880
# 16	173.00	296.00	17.3173	29.6300	70.3700
# 30	241.00	537.00	24.1240	53.7540	46.2460
# 50	328.00	865.00	32.8330	86.5870	13.4130
#100	104.00	967.00	10.4100	96.9970	3.0030
∠#100	30.00	997.00	3.0030	100.0000	0.0000

Fineness Modulus = 3.202

Table 5: Basic Components of Each Mix Design Used

Component		Mix 1	Mix 4	Mix 6
Cement	(lb)	167.10	167.10	167.10
Water	(lb)	67.67	56.62	55.46
Fine Aggregate	(lb)	342.00	336.50	387.31
Coarse Aggregate	(lb)	567.00	579.00	578.00
Air Entrainer	(ml)	140.00	85.00	135.00
Water Reducer (lignosulfonate)	(ml)	-	240.00	-
W/C Ratio*		0.405	0.339	0.332

* W/C Ratio is water cement ratio

Procedure for Data Collection

Since the experiment was going to be replicated once, it was decided that the main set of tests would be run with accelerometers and the replicates would be run without accelerometers. Tests that were run with accelerometers involved making two specimens simultaneously and as identically as practicable, one with accelerometers and one without. The specimen with the accelerometers was then taken apart to retrieve the accelerometers while its twin specimen was saved as representing the particular treatment combination involved. Each experimental unit required a 2.5 cu. ft. volume of concrete, and the size of the concrete mixer was such that only two specimens could be cast per batch.

Preparation of Specimens

Specimen preparation involved several concurrent activities. At the peak of production, eight laboratory assistants were working with the investigator. Specimen preparation usually started with batching a small and slightly wet mix to moisten the mixer before the actual test batch ingredients were charged into the mixer. As the batching operation was continuing, the forms were assembled and oiled, and the rest of the equipment was readied for the desired series of tests. Upon completing the batching, the mixer was charged with the slightly wet batch and mixed for about one minute. It was then discharged before the actual test batch was charged into the mixer. This operation was performed only at the beginning of the day and after lunch or any other extended break period between batches. The test batches were mixed for two minutes and then partially discharged. The

slump and the temperature were immediately taken and recorded. At the same time, the Vebe, the compaction factor, the unit weight, and the air content tests were performed. Prior to the completion of all of these tests, the filling of the two forms to be used for the test was commenced.

Filling the forms was initiated by first setting the reinforcing steel six inches apart and one inch away from the top edge of the form (top edge is the edge form which the vibrator enters the form). The steel used was 5/8 inch diameter deformed bars and each bar was supported on two 4 inches high chairs. One set of forms was used for all of the tests with accelerometers because it was not practicable to move the recording equipment from one position to another.

The form with accelerometer was filled to a height of about one inch before placing the accelerometers at their respective positions. The form was then filled to the brim and the excess concrete screeded off. The filling of the twin form was completed at practically the same time as the form with the accelerometers. As the filling of the two forms to be tested was progressing, four cylinders for compressive strength tests were filled and the pieces of equipment used for testing concrete were readied for the next batch.

As soon as the forms were filled, the generator was started and the frequency was adjusted to the test frequency. The test frequency was checked both by a digital frequency counter and a stroboscope. At the same time, the positions of the accelerometers were adjusted on the recorder to make sure that the traces appeared on the paper. The timer, time lines, paper speed and amplifier gain (if necessary) were then set. When everything was ready (about 10 minutes after mixing),

the concrete was vibrated. Testing was carried out by five people. One person, the frequency reader, recorded the frequency of the vibrator outside the mix and in the mix; one person, the timekeeper, timed the period of vibration; two people, the vibrator pushers, pushed the vibrator into and out of the mix; and another (the investigator) operated the recorder. The procedure for a test will now be described.

As soon as everything was ready, the timekeeper gave the signal for the vibrators to be turned on. The frequency reader then recorded five frequencies(in Hertz) from the frequency counter which displayed the vibration frequency every second. This constituted the in-the-air frequency of the vibrator. As soon as that was completed, the timekeeper gave the signal for the vibrators to be inserted. The vibrators were inserted speedily and the recording of the acceleration was started simultaneously. At about one second prior to the end of testing time, the timekeeper gave another signal and the vibrator pushers would gradually start to bring the vibrator out of the mix at a rate such that when the timekeeper gave another signal to indicate that the testing time was over, the vibrator would just be outside the form. During the whole testing period, the frequency of vibration was continuously recorded from the frequency counter by the frequency reader. As soon as the test was over, he recorded another five frequencies to check the vibrator frequency outside the mix. The vibrators were then turned off.

Because of the speed of recording (160 inches per second) it was not economically feasible to obtain continuous acceleration records throughout each test. A record for the first 2 seconds of vibration

was therefore taken, then another 2-second record was obtained at some randomly-selected time during the first 10 seconds of the period of vibration. In a few instances, such as when test duration was for 5 seconds, the record was taken for the entire test period. Specimens which did not have accelerometers in them were filled together, but were individually tested in accordance with their pre-determined frequencies and durations. Figure 24 shows a typical specimen with accelerometers before testing, and another specimen with its companion specimen vibrated simultaneously.

After testing, the surcharge portion of the form was lowered, the excess concrete was screeded off, and the surface was trowelled. The specimen was then tagged and allowed to set for about half an hour before the forms were stripped and set aside for cleaning. (See Figure 25). As soon as the last specimen was tagged, the next batch of concrete was charged into the mixer and the whole operation repeated until all the specimens for that day, usually six or seven depending on whether they had accelerometers or not, had been made.

Measurement of Nuclear Density and Pulse Velocity

The measurement of the nuclear density was carried out under two different conditions. These were the plastic condition and the condition after a 7-day moist curing. The density measurements on the plastic specimens were taken about 30 minutes after the specimens were cast. Two types of density measurements were carried out on the cured specimens, namely above and below the steel. Readings were taken on the surface in the plastic condition at each of the five positions shown in Figure 26. After moist curing, density readings

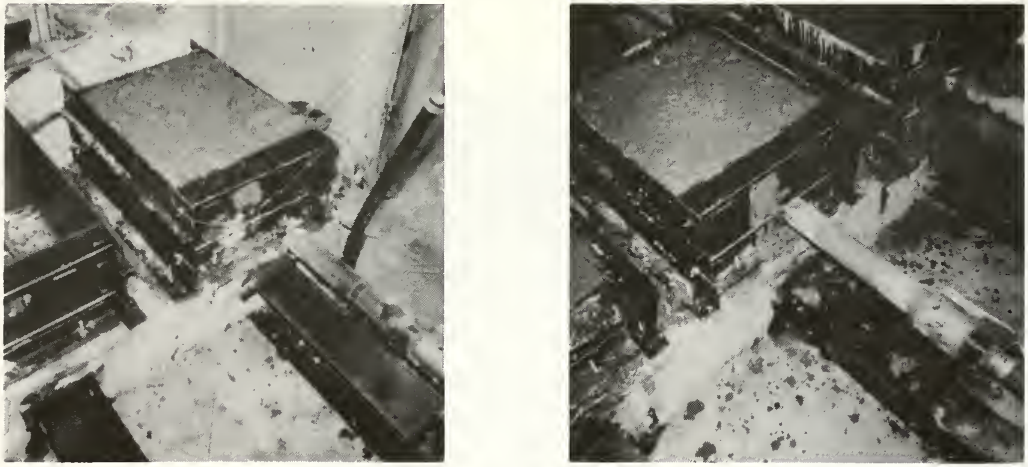


Figure 24

Typical Specimen with Accelerometers before Vibration and Companion Specimens after Vibration

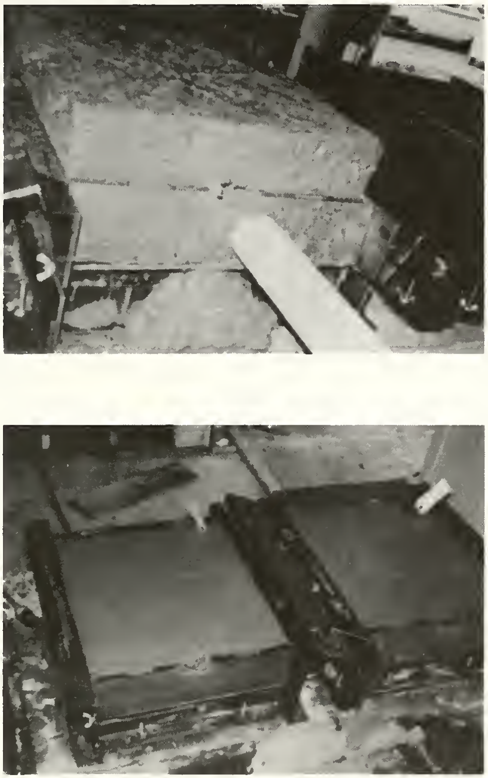


Figure 25

The Lowered Surcharge Portion of the Form and Some Finished Specimens

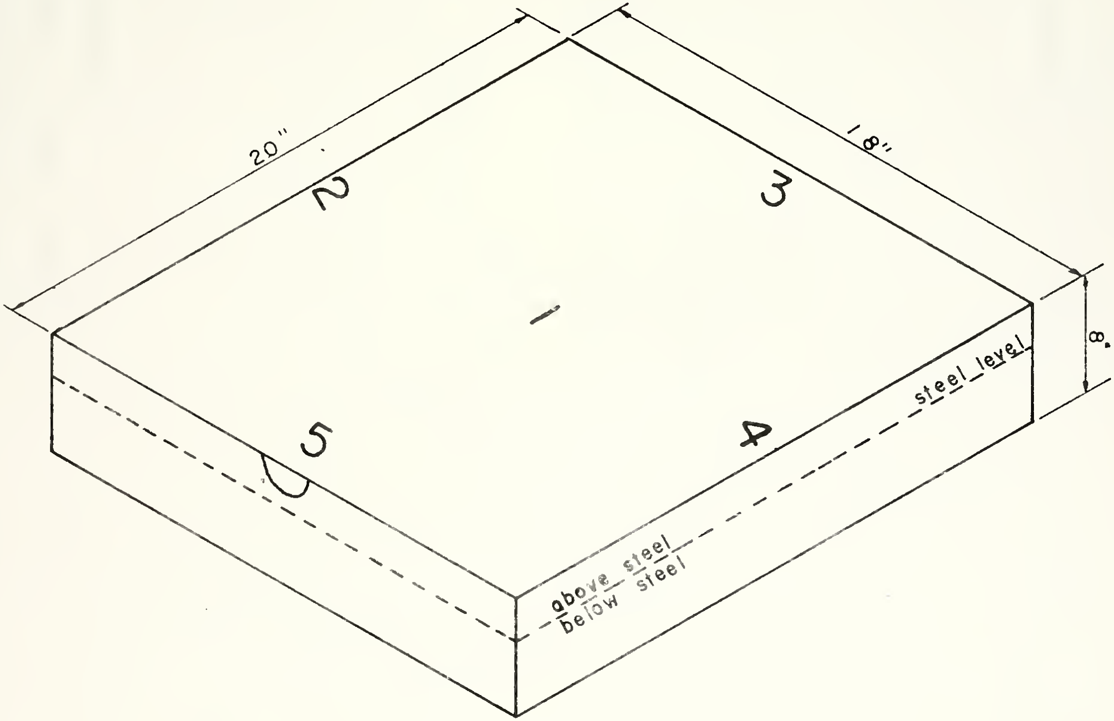


Figure 26

Positions of Density Readings on the Specimens

were taken at those same five positions for the above-steel readings. The specimen was then turned over and five below-steel readings were taken at the corresponding positions on the opposite face.

The pulse velocity measurements were taken on the same day as the cured readings of the nuclear density. Three conditions of pulse velocity were measured, all taken after curing. The readings were a) across-the-depth, b) above-the-steel and c) below-the-steel. The across readings were taken across opposite faces of the 8 inch side and at the five positions where the nuclear density readings were taken. The above-the-steel readings were taken at the six indicated positions in Figure 27, three along each side. The below-the-steel readings were taken at the six indicated positions in Figure 27. All the specimens cast on the same day were tested together for pulse velocity, nuclear density and 7-day compressive strength.

Measurement of Amplitude

The measurement of vibrator amplitude turned out to be extremely difficult. Initially, it was intended that the displacement amplitude be measured by attaching accelerometers to the tip of the vibrator and measuring the acceleration. The acceleration could then be integrated twice to obtain the displacement. To this end, three accelerometers were attached to the vibrator as shown in Figure 28. The accelerometer records thus obtained were satisfactory for the 7,200 cpm frequency but it was discovered that the accelerometers were erratic at higher frequencies. A visual examination which indicated that the crystals were probably cracked was confirmed when the accelerometers were put on a shaker. The manufacturers of the accelerometers

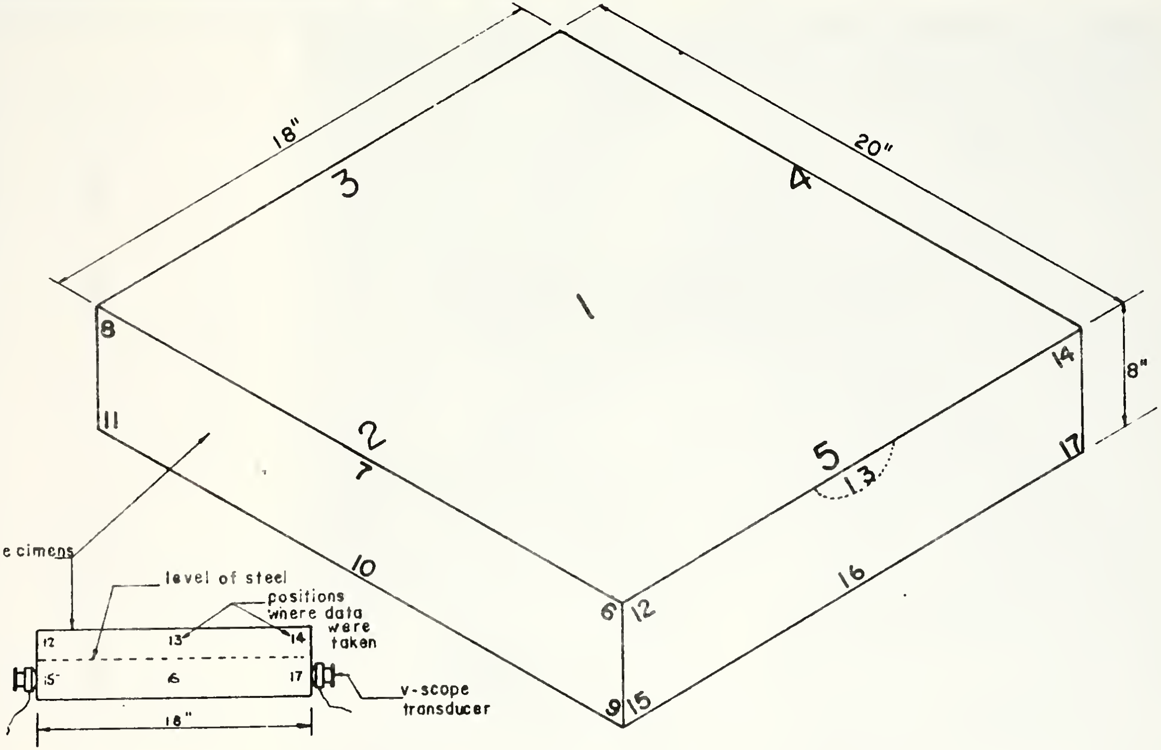


Figure 27

Positions of Pulse Velocity Readings on the Specimens

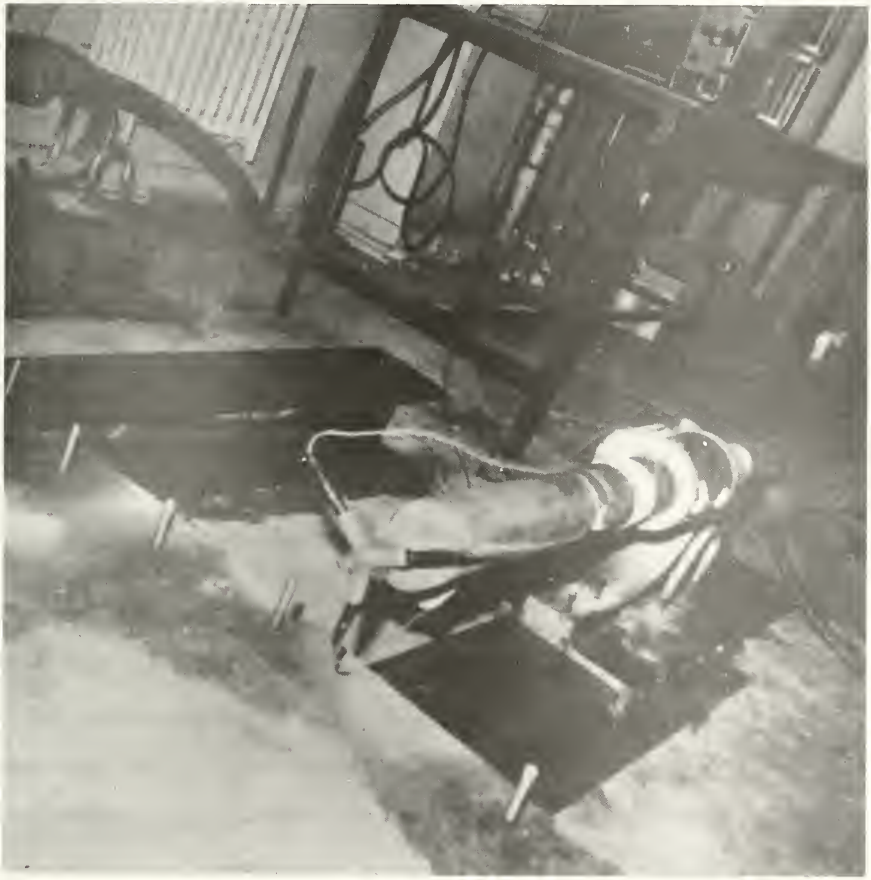


Figure 28

Triaxial Mounting of Accelerometers

were contacted and a "stronger" set of accelerometers was forwarded. However, when attempts to measure amplitude with this new set also met with failure, it was decided to measure the amplitude with an LVDT. Figure 29 shows the schematic of the LVDT connection to the vibrator. The measurement of displacement (amplitude) proceeded as with the measurement of acceleration, but was confined to Mix 6 only. Before taking any readings in the mix, displacement measurements were taken with the vibrator operating in the air at two frequencies of 7,200 and 13,200 cpm. These measurements were taken at different positions along the vibrator to determine whether the LVDT had enough range for the displacements to be encountered within these test frequency limits.

Measurements of displacements in the mix were taken at three positions along the vibrator. The first and the second positions were spaced 6 inches apart whilst the second and the third were 4½ inches apart. The procedure was to position the vibrator as shown in Figure 30 with the sleeve for the LVDT around it. Concrete was then placed around the vibrator and the LVDT stem until the form was filled to the brim. Testing was carried out as described previously except that the time was held at approximately 7 seconds for all the frequencies examined. After testing had been completed, the form was emptied, the LVDT stem removed and cleaned, the form cleaned and set up for the next position. Another batch of concrete was then mixed, the form was again recharged and the measurement for the second position was taken. This was continued to the third position for each of the five frequencies of vibration used.

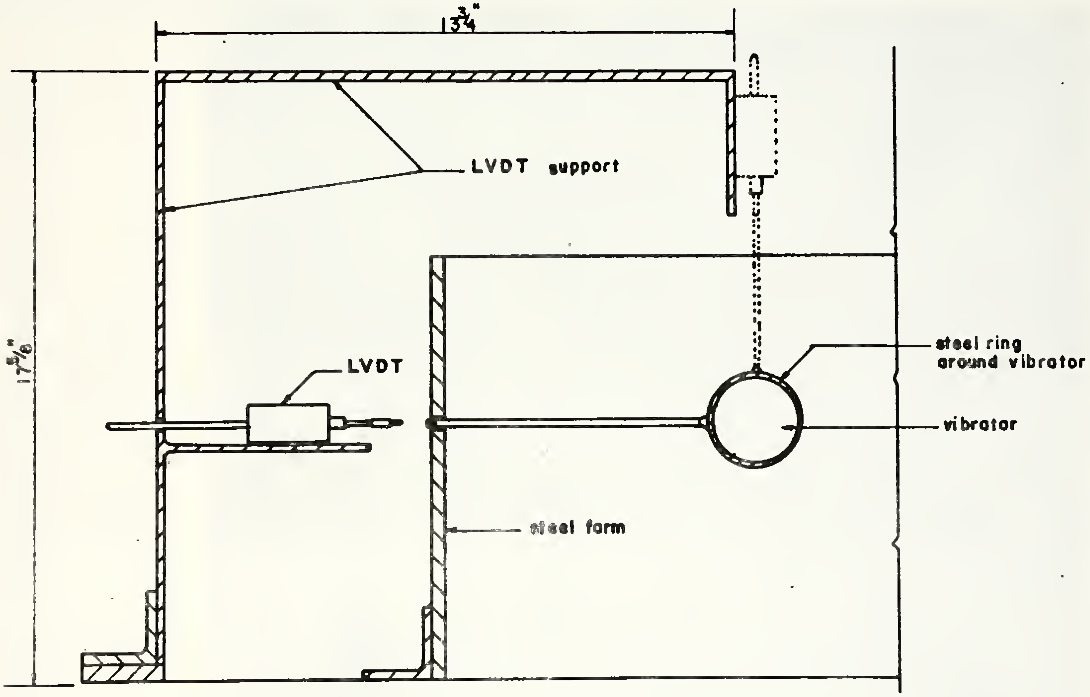


Figure 29

Schematic of LVDT Connection to Vibrator

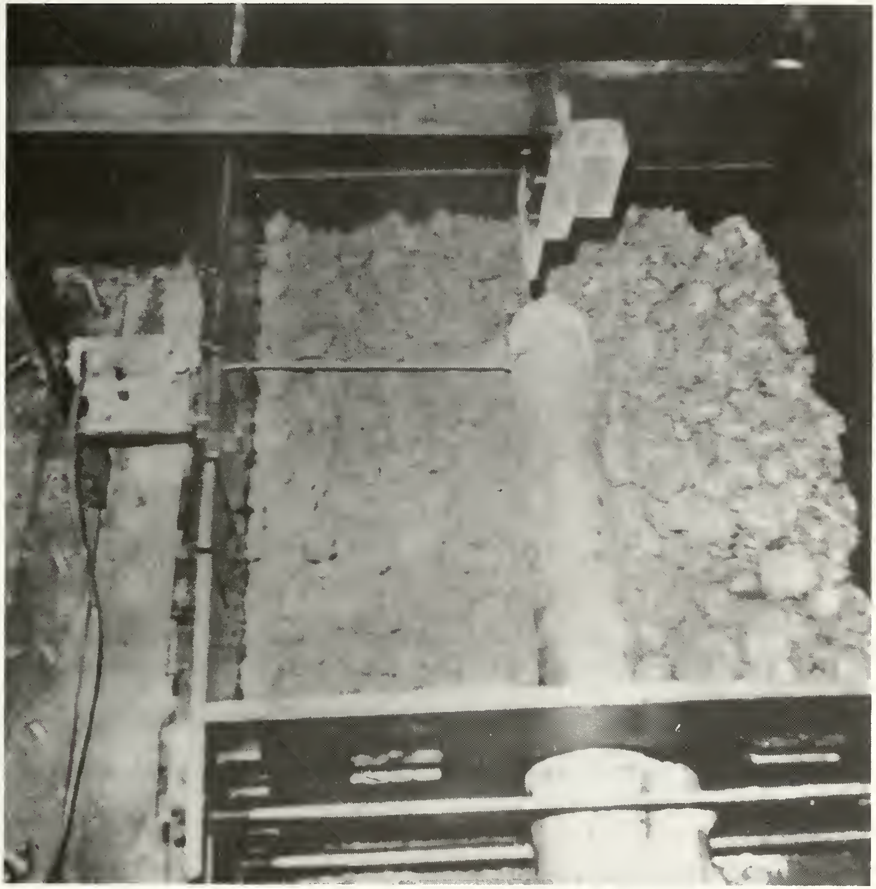


Figure 30

LVDT Attachment to Vibrator at Position One

Comments on Testing Program

The testing program did not proceed entirely smoothly. Several problems arose with the electronic equipment, especially with the recorder. This latter item was late in delivery, and it was also plagued with frequent breakdowns. Labor difficulties were also encountered, particularly so in the latter part of the summer when the available labor was insufficient to perform all of the desired tests within an acceptable time period. Tests such as Vebe, compaction factor, air content and unit weight were eliminated under those circumstances, but were restored as soon as sufficient labor was available.

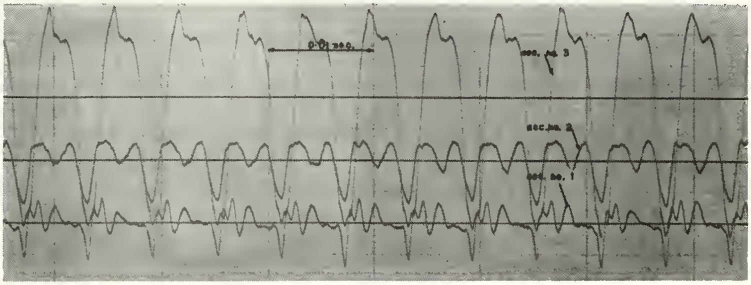
Data Analysis

Reduction of Data

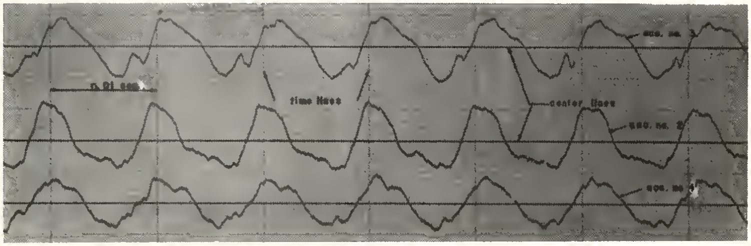
The voluminous data gathered during this experiment dictated an analysis by computer. The first step was to reduce the nuclear density and pulse velocity data to pounds per cubic foot and feet per second, respectively.

As previously discussed, the nuclear density data was analyzed by referring to a standardized curve which had been fitted to readings taken on specimens of predetermined densities. The pulse velocity was computed from the basic relationship that the velocity equals the distance or displacement divided by time. Computer programs for these analyses are supplied in Appendices B and C respectively.

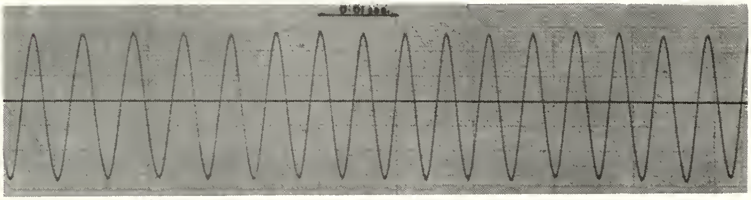
The reduction of the acceleration and amplitude data was more difficult. Figure 31 shows typical acceleration and displacement



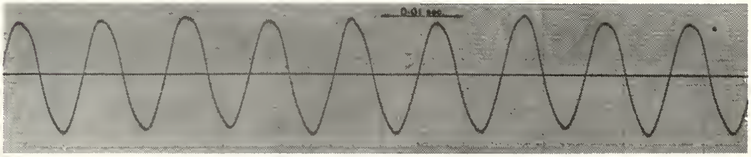
Acceleration records at positions 1, 2 and 3 for 12000 cpm.



Acceleration records at positions 1, 2 and 3 for 7200 cpm.



Displacement record at the tip of vibrator for 12000 cpm.



Displacement record at the tip of vibrator for 7200 cpm.

Figure 31

Typical Acceleration and Displacement Records

records. Initially, the output was visually examined and the frequency checked to see how well it corresponded with the input frequency. Then three cycles which were believed to be representative of the record were selected and completely digitized on a LARR-V digitizer for computer analysis. A computer program to analyze the curves by Fourier curve fit was developed. Two available routines in the Purdue University Computer Library (RUNLIB) were incorporated into this program. The first program, FORIT (herein named FORAT), was for Fourier analysis. The second, SETPLT, was for plotting the input points, the Fourier points and all other plotting that was needed in the analysis. The computer program which was developed as a part of this research also plotted the Fourier Line Spectrum, the plot of the power at a harmonic versus the frequency of the harmonic. This plot made it possible to identify the most powerful harmonic of the ten harmonics which were determined from the Fourier analysis (41). The remainder of the vibration analysis was then based on the most powerful harmonic, which in this case was the first harmonic. Appendix D shows the program and its associated library routines. DISPLA, a subroutine to this program, was used to perform a similar type of analysis on the amplitude data, except that there was no need to plot the Fourier Line Spectrum since the curve was obviously sinusoidal.

Statistical Analysis

The initial objectives of this experiment were, firstly, to determine the frequency or frequencies which gave the best compaction as measured by the highest density, pulse velocity, and acceleration; and, secondly, to see which parameters seemed to have the greatest

influence on the compaction achieved. It was apparent that the validity of the results could best be ascertained by statistical analysis.

Analysis of Nuclear Density Data

It will be recalled that the statistical design was a composite one with multiple factors. After the assembly of the of the data, five factors were identified, namely replication (R) with 2 levels; mixes (M) with 3 levels; treatments or frequency-duration combination (T) with 9 levels; conditions (C) with 3 levels i.e. plastic, above-steel, and below-steel; and positions (P) with 5 levels. See Figure 26 for the five positions.

Before any analysis could be carried out, it was necessary to check the various assumptions on which each of the different analyses was based. For analysis of variance (ANOVA), it is imperative to test for the homogeneity of variances since the ANOVA technique is based on this assumption (33). The test for homogeneity was the Foster-Burr Test which is a part of a Statistical Library Program at Purdue University. This program is shown in Appendix E. Since there was only one observation per cell, this test could not be run on the total data. Instead, the center point in the composite design was used as the reference datum. The test was run on 90 observations and showed a Q statistic of $0.0756 > 0.042$. This indicated that the assumption of homogeneity of variances was false. Visual observation of the cell means and variances indicated that there was a negative correlation between the means and variances. This suggested a data transformation of the general form:

$$Y_i = \sqrt{B} - \sqrt{B - y_i} \qquad (33)$$

where:

Y_i = transformed observation for each i

B = highest value observed for y

y_i = individual i observation, here i = 1 ... 90.

When this transformation had been performed, the homogeneity test was again run and the and the variances were found to be homogeneous (Q statistic was $0.0329 < 0.0420$). The other usual assumptions for ANOVA are that a) each observation is random, b) the design model is additive and c) the observations are normally distributed. Since homogeneity was achieved, it was not deemed necessary to test the rest of these assumptions.

Following the homogeneity test, a series of ANOVA tests were run on the transformed data. Separate analyses were performed on the total data, on separate mixes, on separate paths, on separate treatments above and below steel, and on separate conditions. The test in each case was run depending on the factors or interactions that turned out to be significant in the preceding analysis. The result of one such run is shown in Appendix F and typical input programs are shown in Appendix G.

The statistical model was a split plot design, but the computer programs used analyzed the data as completely randomized factorial design; therefore, the computer output had to be arranged to reflect the restrictions on randomization. This is illustrated in Table 6. Since there was only one observation per cell, the statistical

Table 6: Arranged Computer Printout for Split-Split Plot Design

ANOVA

Source	Degree of Freedom	Mean Square
R	1	4.2577
$\delta(i)$	0	0.0000
M	2	5.4808
RM	2	0.8390
$\lambda(ij)$	0	0.0000
T	8	2.8314
RT	8	2.1156
MT	16	0.9342
RMT	16	0.9894
$\eta(ijk)$	0	0.0000
C	2	194.6493
RC	2	1.6883
MC	4	0.2381
TC	16	0.6350
RMC	4	0.5137
RTC	16	1.2332
MTC	32	1.6932
RMTC	32	0.6950
$\varepsilon(ijkl)$	0	0.0000

Table 6, cont.

P	4	3.1404
RP	4	0.1423
MP	8	0.5732
TP	32	0.1309
CP	8	0.9129
RMP	8	0.1427
RTP	32	0.1089
MTP	64	0.1649
RCP	8	0.1756
MCP	16	0.1592
TCP	64	0.0928
RMTP	64	0.1623
RMCP	16	0.0760
RTCP	64	0.1086
MTCP	128	0.1088
RMTCP	128	0.1472
$\varepsilon(ijklm)$	0	0.0000

analysis did not yield a residual error term which could be used to test the significance of factors and interactions. For this reason, it was necessary to combine selected interaction variances into a pooled error term which could then be used to test the remaining interactions and factors. Appendix F shows that the 4-way and higher-order interactions, except for RMTC, were very small. Most of the 3-way interactions were also very small. It was therefore decided that all of these terms would be pooled if they were found to be insignificant at $\alpha = 0.25$. This calculation was performed and the significant effects were recorded and interpreted. Appendix H shows a sample calculation for the pooling of interaction variances shown in Table 6.

Although a basic objective of this experiment was to determine the most effective treatment combination, the various ANOVA failed to identify treatment as a significant factor. It was then decided to run a sequential range test on the actual means of each treatment combination for each mix. The sequential range test, or Newman-Keul's Test as it is sometimes called, is a method of ranking the means of various factors based on the number of observations and the mean square of each factor or factor level. The nine treatment combinations which were ranked are shown on page 37. Other analyses carried out on the nuclear density data will be discussed in a later section.

Analysis of Pulse Velocity Data

The analysis of the pulse velocity data was similar to that which was performed on the nuclear density data. As noted earlier, it was necessary to perform the same basic type of data transformation in order to achieve homogeneity of variance. However, in the pulse

velocity analysis only four factors were identified, namely, the replication (R), the mix (M), the treatment (T) and the paths (P) which had 17 levels. Table 7 summarizes the ANOVA tests carried out on both nuclear density and pulse velocity. It also shows the significant factors. In this table, the factors and interactions are explained as follows: mixes (M), treatments (T), conditions (C); for pulse velocity only, conditions (C) describe the pulse velocity across the depth, above and below the steel, and paths (P); for nuclear density, conditions (C) describe the ~~densities~~ densities in the plastic state, above and below the steel, and positions (P). Interactions are described by a combination of two or more of the factors mentioned above. For example, the interaction of mixes and treatments is MT and of mixes, paths and conditions for pulse velocity is MPC, etc.

Other Analyses

The completion of Newman-Keul's Test on the densities, pulse velocity and acceleration data led to the conclusion that there were basic relationships between some of the variables examined. To examine these relationships, correlation and regression analyses were run on all of the variables identified. The computer programs used for these analyses were SPSS 15 and SPSS 21. SPSS 15 was used for stepwise regression and SPSS21 for nonlinear regression. Both programs are available in Purdue's Statistical Package for the Social Sciences Library. The variables examined were the input frequency (FI); slump (SL); compressive strength (CS); amplitude (AMPL); temperature (TE); acceleration (ACCL); densities in plastic condition (DP), above the steel (DA) and below the steel (DB); pulse velocities above the

Table 7: Summary of ANOVA Results

Tests	Significant Factors and/or Interactions			
	Path No.	Pulse Velocity	Position No.	Nuclear Density
1. Total Data		P,TP,MTP		P,MP,C,MTC
2. Separate Mixes:				
a) Mix 1		TP		P
b) Mix 4		P		P
c) Mix 6		T,P,TP		P,TP
3. Separate Paths:				
a) Surface	1	M	1	None
i. across depth for pulse velocity	2	None	2	"
	3	"	3	"
ii. plastic for nuclear density	4	"	4	"
	5	M	5	"
b) Above Steel	6	None	6	None
	7	"	7	"
	8	"	8	"
	9	"	9	"
	10	"	10	"
	11	"		
c) Below Steel	12	MT	11	M
	13	MT	12	None
	14	MT	13	MT
	15	M	14	MT
	16	None	15	None
	17	M,MT		
4. Separate Frequencies and Times:				
a) 9600; Above & Below Steel		P,MP,MTC		C,TC,MP
b) 10800; "		C		C,P,TC,MTP
c) 12000; "		C, MTC		P,MTC
5. Separate Conditions:				
a) Depth/Plastic		M,P		None
b) Above Steel		P,MP		P,MP
c) Below Steel		P,MP		M,P,MT,MP
d) Above & Below Steel Together		MC,MTC,P,MP,CP,MCP		P,MP,MTC

steel (PVAS), and below steel (PVBS); peak to peak acceleration (PPA), and response frequency (RF). These variables represented the vibrator and the concrete parameters input to the experimental units before vibration or parameters measured after vibration. The discovery of 98% correlation between FI and RF, 96% between AMPL and RF, 96% between PVAS and PVBS, and 94% between ACCL and PPA led to the decision to plot these relationships and to run a linear regression analysis through the various points. Multiple regression was then run to relate the dependent variables such as the density below steel, pulse velocity below steel, and amplitude with such variables as slump, input frequency, temperature (of concrete at placement), density above steel, pulse velocity above steel, etc., in order to determine which independent variables best described any of those dependent variables. The analysis was carried out by stepwise regression and no attempt was made to force any particular independent variables into the regression model. Care was taken not to include two highly correlated variables on the same side of a regression model in order that the effects of the other independent variables on the dependent variables would not be masked by the highly correlated variables. Appendix K shows a typical printout from the SPSS 15 program.

RESULTS OF DATA ANALYSIS

The analysis indicated that the factors of major interest were the treatments (T), the mixes (M), and the conditions (C). Table 7 shows the results of ANOVA tests on the transformed data for pulse velocity and nuclear density. At this point, it is believed that a restatement of the variables involved will aid in reading the table. For both the pulse velocity and nuclear density, mixes and treatments are represented as indicated above. For pulse velocity, the other parameters are paths (P) and conditions (C) which describe the velocities across the depth, above and below the steel. For nuclear density, the only different parameters are the positions (P) and the conditions (C) which describe the densities in the plastic state and the cured state, above and below the steel. There usually are 2- or 3-level interactions which turn out significant. An example is MT which is a 2-level interaction of treatments and mixes. This can be continued for 3-, 4-, etc. level interactions of the main effects.

Results from Analysis of Variance

From Table 7, it is seen that paths, positions, conditions, treatment-paths, mix-positions, condition-positions, and mix-conditions are significantly different for both the nuclear density and pulse velocity. The significance of the 3-level interactions MTP and MTC seemed interesting. In the next analysis when the mixes were

separated it was found that treatments, paths and positions plus a number of their interactions were significant. This was especially true in Mix 6 which, it will be recalled, was the stiffest mix design tested. Separating the paths showed treatments were not significant above steel but the mix-treatment interactions were significant for both the nuclear density and pulse velocity. Analyses 4 and 5 did not contribute further information since it was already known that, holding all of our mixes constant, there were differences between the various treatments imparted on the concrete, It was not possible, however, to tell whether any treatment was significantly better in compacting concrete than the others.

Results from Newman-Keul's Test

The treatment which gave the highest pulse velocity, nuclear density and acceleration was identified by Newman-Keul's Test. Tables 8 and 9 are summaries of the rankings. For Table 8, nine treatment combinations are listed. Below is a list of the original order of these treatments before ranking.

Original Order	Treatment Combination
1	7200 cpm for 9 seconds
2	9600 cpm for 7 seconds
3	9600 opm for 11 seconds
4	10800 cpm for 5 seconds
5	10800 cpm for 9 seconds
6	10800 opm for 13 seconds
7	12000 cpm for 7 seconds
8	12000 cpm for 11 seconds
9	13200 cpm for 9 seconds

Table 8: Rank of Means by Newman-Keul's Test for 9 Treatment Combinations for All Mixes.

	Pulse Velocity			Nuclear Density			ACCL	PPA
	Ranked Order of the Original Treatment Numbers							
Rank No.	Depth	Above Steel	Below Steel	Plastic	Above Steel	Below Steel		
1	7	7	6	7	7	7	8	5
2	4	6	7	8	8	5	6	6
3	6	4	9	5	6	8	5	9
4	5	9	4	6	4	4	9	8
5	8	8	8	9	5	9	7	7
6	2	5	5	4	1	6	4	4
7	9	2	2	3	3	2	2	2
8	1	3	3	1	2	3	3	3
9	3	1	1	2	9	1	1	1

Table 9: Rank of Means by Newman-Keul's Test for 5 Frequencies For All Mixes.

	Pulse Velocity			Nuclear Density			ACCL	PPA
	Ranked Order of the Original Treatment Numbers							
Rank No.	Depth	Above Steel	Below Steel	Plastic	Above Steel	Below Steel		
1	3	4	4	4	4	4	5	5
2	4	5	3	3	3	3	3	3
3	2	3	5	5	1	5	4	4
4	5	2	2	1	2	2	2	2
5	1	1	1	2	5	1	1	1

For Table 9, only frequencies were examined. These were originally originally ordered as follows; 1) 7,200, 2) 9,600, 3) 10,800, 4) 12,000, and 5) 13,200 cpm. The computer program after ranking the means also tested the differences between the means at 1% and 5% levels of significance. See Appendix J for a typical Newman-Keul's Test program and computer printout.

The overall finding of the analysis was that the 12,000 cpm frequency applied for 7 seconds produced the greatest effect in compacting concrete for both the nuclear density and the pulse velocity; there was no clear cut indication in the case of the acceleration. In all of the analyses, there was no significant difference between the effects of the 12,000, 10,800, and 13,200 cpm frequencies although each of these frequencies produced significantly higher effects than the 9,600 and 7,200 frequencies at either 1% or 5% levels. The 9,600 frequency was only slightly better than the 7,200 frequency.

Results from Regression Analysis

The plots of input frequency to response frequency indicated a frequency reduction of about 15% for each of the mixes, but the relationship was linear in each case. A regression analysis run on the data with the response frequency (RF) as the dependent variable (measured by the accelerometers) showed an equation of the type:

$$RF = 0.7723\ FI + 685.563 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The residual mean for the analysis was 1.51×10^{-11} which is essentially zero; the R^2 value was 96%; and the F value was 599.868 which was significant at 0.005 level. A confidence band was placed on

the slope b = 0.7723 at 0.005 level of significance. The slope then was:

$$b = 0.7723 \pm 0.0878 \quad \text{..........................(3)}$$

The second through the fifth order models were also examined. The second order model had a partial F value of 3.291 which was significant at 10% level; however, the analysis contributed only 0.5% to the R^2 value. The third and fifth order models could not enter into the regression model because of low F values. The fourth order variable had an F value less than one and contributed 0.1% to the R^2 value. These analyses showed that the linear model adequately represented the relationship between the input and the response frequencies. The entry of other variables such as slump, temperature, acceleration and compressive strength into the regression model did not alter the conclusion to any appreciable extent.

The regression of density below steel (DB) with FI, slump (SL), compressive strength (CS), acceleration (ACCL) explained only 39% of the variability, but SL and CS contributed significantly to explaining the variability in DB. Including DA, density above steel, increased R^2 to 68% with an overall F value of 8.8 which was significant at 0.005% level. Including temperature (TE) increased the R^2 value to 68.6% and lowered the F value to 7.2 which was still significant at 0.005%. The equation developed was:

$$\begin{aligned} DB = {} & 5.253\ SL - 0.699\ DA + 0.0054\ CS + 0.00039\ FI + \\ & 0.25\ ACCL + 0.224\ TE + 194.89 \quad \text{.............. (4)} \end{aligned}$$

where:

DB = density below steel [pounds per cubic feet (pcf)]

SL = concrete slump (ins.)

CS = 28-day compressive strength [pounds per square inch (psi)]

FI = input frequency (cpm)

ACCL = acceleration (g's)

TE = concrete temperature at placement ($^{\circ}$F)

However, since ACCL and TE did not contribute significantly to explaining the variability (F values less than 1.0), they were eliminated from the equation. The new model with F level significant at 0.5% was:

$$DB = 4.980\ SL - 0.635\ DA + 0.005\ CS + 0.00053\ FI + 204.17 \quad \ldots(4)$$

With this equation, the density below steel can be predicted with 99.5% confidence [illegible] the data from this experiment.

A similar analysis for pulse velocity showed the relationship between PVAS and PVBS as follows:

$$PVBS = 1.463\ PVAS + 0.042\ FI + 241.67\ SL + 0.084\ CS - 7960.10 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where:

PVBS = pulse velocity below steel [feet per second (fps)]

PVAS = pulse velocity above steel (fps)

FI, SL, and CS are as previously described

However, only PVAS contrbuted significantly at 5% level to explaining the variability.

The attempt to predict the acceleration from amplitude, slump and input frequencies gave an equation of the type shown below with an overall F value of 8.92 which was significant at 0.005% level.

$$\text{ACCL} = 4772.88\ \text{AMPL} + 1.423\ \text{SL} - 0.00105\ \text{FI} - 7.843 \quad \ldots\ (6)$$

where:

AMPL = amplitude (ins)

ACCL, SL, and FI are as previously described.

However, the R^2 value was only 53.8% which was considered very low.

The bivariate relationships examined are reported below. The relationship between amplitude and frequency, upon plotting, appeared tenuous since many regression models could be run through the points.

$$\text{AMPL} = 3.067 \times 10^{-7}\ \text{FI} + 0.00107; \quad R^2 = 93.4\% \quad \ldots\ldots\ldots\ (7)$$

$$\text{PVBS} = 1.694\ \text{PVAS} - 10073.94; \quad R^2 = 92.9\% \quad \ldots\ldots\ldots\ (8)$$

$$\text{ACCL} = 0.283\ \text{PPA} + 0.093; \quad R^2 = 89.2\% \quad \ldots\ldots\ldots\ (9)$$

where all parameters are as previously explained.

Summary of Results

The analyses of the data indicated the following results:

1. A vibration frequency of 12,000 cpm operated for a duration of 7 seconds gave the highest means for both the pulse velocity and nuclear density.
2. In terms of frequencies, there was no significant difference between the compaction abilities of the 12,000, 10,800, or

13,200 cpm as a group. However, they all produced significantly better results than both 9,600 and 7,200 cpm at 5% and/or 1% levels. The 9,600 cpm was only slightly more effective in compacting concrete than the 7,200 cpm.

3. There was no significant difference at 5% level between the density above the steel measured in the plastic state and after curing for 7 days.
4. The density above the steel was, at 5% level, significantly higher than below the steel. On the average, the density above the steel was almost 7% higher than below the steel for each of the 3 mixes tested (6.6% for Mix 1, 6.7% for Mix 4 and 6.4% for Mix 6). The smallest and the largest differences occured in Mix 6 at 13,200 and 9,600 cpm respectively. These values were -7.94% and 14.13% respectively. See Table 10.
5. The input frequency was more than 15% higher than the response frequency picked up by the accelerometers in the mix (15.04% in Mix 1, 15.13% in Mix 4, and 17.16% in Mix 6).
6. The relationship between the input and the response frequencies was linear for all the 3 mixes investigated, and therefore could be predicted by an equation.
7. Slump and age of specimen within the range at which the testing occured (7 to 9 days) in this experiment did not significantly affect the pulse velocity measured, but slump significantly affected the nuclear density measured at 5% level.
8. From the multiple regression analysis, the variability in density could be explained more in terms of concrete parameters such as slump and compressive strength than in terms of vibrator

parameters such as frequency and amplitude. However, various other factors are necessary to explain this variability to an appreciably reliable level. Such variables may include the air content, the viscosity of the paste, and possibly the speed of paving.

9. Frequency and amplitude of vibration do not seem to be as critical to vibratory compaction of concrete as was conjectured by several earlier investigators.

Table 10: Differences Between Nuclear Density Above and Below Steel in Percent.

Treatment Combination	Differences		
	Mix 1	Mix 4	Mix 6
7200 cpm for 9 seconds	5.79070	6.24559	11.65655
9600 cpm for 7 "	6.35781	7.02468	6.29441
9600 cpm for 11 "	2.04030	5.00319	14.13046
10800 cpm for 5 "	6.65590	7.55817	6.34638
10800 cpm for 9 "	5.89166	6.88244	7.24290
10800 cpm for 13 "	9.82209	7.07406	6.06603
12000 cpm for 7 "	7.84415	4.28489	5.71808
12000 cpm for 11 "	7.73458	6.35578	7.79961
13200 cpm for 9 "	7.57672	10.23906	- 7.94140
Mean of Differences (%)	6.63488	6.74087	6.36811

DISCUSSION OF RESULTS

The fact that the frequency of 12,000 cpm applied for 7 seconds topped all the other frequencies in its effect on the compaction of concrete merely corroborated the findings of Las Forssblad (10), Wadhwa (22), and of many other investigators who proposed the existence of a frequency that produced "remarkable" effects on the strength of concrete and cement pastes. However, the results of this experiment also showed that there was no significant difference between the compaction effects produced while vibrating in a range between 10,800 and 13,200 cpm. The fact that the 13,200 cpm frequency did not produce a markedly higher density than the lower frequencies of 10,800 and 12,000 cpm is of interest. The consequent reduction in the life of the eccentric bearings when vibrators are operated at 13,200 cpm or higher frequencies, coupled with the high levels of noise, make it economically unjustifiable to operate vibrators at such speeds in highway construction.

It has long been recognized that mechanical aids are required to compact harsh-dry mixes. Vibration frequency, however, did not seem to be a particularly critical factor in the compaction of concrete as indicated by the results of this experiment. It was found to have a very low correlation with the compaction density (29% with DB, 11.4% with DA, and 51% with DP), which confirms the earlier findings of Sullivan (12) and Bower, et al. (27).

The differences between the input frequency and the response frequency created some concern during data analysis. Although it was was expected that the input and the response frequencies would be equal or nearly equal, a 15% reduction was actually encountered (17% in Mix 6, see Table 11). Efforts made to explain this discrepancy did not adequately satisfy the investigator's curiosity. It appears that the activity at the interface between the vibrator and the concrete involves a more complicated phenomenon than "coupling", or a lack of it, as suggested by the literature review. This phenomenon has been left to future researchers to investigate. It should be noted, however, that in all of the mixes tested the relationship between the input frequency and the response frequency was linear (see p. 93).

The literature review identified various other investigators such as Bennett and Gokhale (16), Green (17) and Davies (18) who have viewed acceleration as the most significant parameter affecting concrete compaction by vibration. Cussens and Plowman (19, 20), however, indicated that acceleration as measured experimentally in clamped molds might not be applicable to the larger and more stable masses which were vibrated in the field. Kirkham and Whiffin (24), using surface vibrators in the field, concluded that acceleration had no effect on the achieved compaction. The findings of this experimental study reported herein support the conclusion of the latter investigators. It is believed that acceleration has only a minimal effect, or none at all, on the compaction of concrete for highway paving.

Kolek (8) showed that amplitude is related to frequency. The acceleration of a particle is also a function of the amplitude. Studies of surface vibration have concluded that amplitude, frequency

Table 11: Input and Response Frequencies for All Mixes, and the Differences.

Mix 1			Mix 4			Mix 6		
Input Frequency (cpm)	Response Frequency (cpm)	Difference (%)	Input Frequency (cpm)	Response Frequency (cpm)	Difference (%)	Input Frequency (cpm)	Response Frequency (cpm)	Difference (%)
6840.000	5987.927	12.45720	6864.000	5921.148	13.73619	6528.000	5191.807	20.46864
8940.000	8410.256	5.92554	8145.000	7125.661	12.51490	8040.000	6633.167	17.49792
8845.714	7710.957	12.82833	8600.000	7521.796	12.53726	8208.000	6703.625	18.32815
10780.000	8901.480	17.42597	10740.000	9258.587	13.79342	10840.000	8860.490	18.26117
10886.000	8836.598	18.82604	10837.000	9193.186	15.16853	11267.333	9768.336	13.30392
10868.571	9575.446	11.89784	10815.000	9202.606	14.90887	10750.000	8936.588	16.86895
12084.000	9810.096	18.81747	12276.000	10214.124	16.79599	13188.000	10577.975	19.79091
12105.000	9983.844	17.52297	12120.000	9961.918	17.80596	12860.000	10344.623	19.55970
13210.000	10606.100	19.71158	13670.000	11083.770	18.91902	12735.000	11419.696	10.32826
Mean of Differences (%)		15.04588			15.13113			17.15640

and force are the most significant parameters affecting concrete compaction. The experiment reported herein found that amplitude was highly correlated with the input frequency (correlation coefficient of 96.7%) which confirms the conclusion reached by Kolek. On this basis, it is concluded that the effect of amplitude on concrete compaction, like that of frequency, is very minor.

The prediction equations identify slump as a large contributing factor to the measured density below steel, and therefore to the amount of compaction achieved. It was noted on page 98 that the largest variation in density occured in Mix 6, the stiffest mix of those examined. Table 11 shows that this mix also had the largest mean difference between the input and the response frequencies. Finally, this mix had the largest reduction in input amplitude as measured with the LVDT and in the response amplitude as calculated from double integration of the acceleration value (see Table 12). All of the above statements point to the importance of slump in the vibratory compaction of reinforced concrete. Table 13 shows the measured slumps for the specimens tested and reported in this analysis. From these results of this experiment, it is concluded that concrete slumps of less than 1" will generally introduce greater variability in vibratory compaction of concrete than will higher slumps. Further, at such low slumps, the concrete might not be getting adequate compaction. These conclusions support the findings of Bower and Gerhardt (27).

Investigations of other concrete parameters, such as temperature and 28-day compressive strength (see page 95) showed that temperature did not contribute significantly, whereas, compressive strength

Table 12: Variation of Input Amplitude with the Response Amplitude

Theoretical Frequency (cpm)	Input Amplitude (ins.)	Mix 1	Mix 4	Mix 6
		Response Amplitudes (ins.)		
7200.000	0.00297	0.00238	0.00276	0.00284
9600.000	0.00364	0.00170	0.00264	0.00185
10800.000	0.00459	0.00376	0.00351	0.00355
12000.000	0.00475	0.00356	0.00338	0.00104
13200.000	0.00509	0.00331	0.00301	0.00120

Table 13: Slumps and Corresponding Theoretical Input Treatments for The Specimens Tested.

Theoretical Input Treatments		Slump (ins)		
Frequency (cpm)	Duration (secs)	Mix 1	Mix 4	Mix 6
7200	9	2.00	1.25	0.50
9600	7	1.50	1.25	0.50
9600	11	2.00	2.00	0.25
10800	5	1.50	1.50	1.50
10800	9	2.00	1.75	0.75
10800	13	1.75	2.25	1.25
12000	7	1.75	1.75	1.75
12000	11	1.50	2.50	1.25
13200	9	1.75	1.25	0.75

did affect compaction significantly. Normally, compressive strengths obtained from cores taken from the test specimens would have been utilized in the regression model instead of the 28-day strength obtained from the test cylinders. However, time and budget did not permit taking of oores from the tested specimens. Insufficient information was obtained on air content and therefore its influence on the regression model could not be ascertained. The amount of information obtained about slump and strength, and the analyses of frequency, amplitude and acceleration tended to support the findings of Kolek (8), Forssblad (10), and Bower and Gerhardt (27) that the physical concrete parameters influenced compaction more than the vibrator parameters.

The results of this experiment suggest the density below the steel can, with at least 95% confidence level ($R^2 = 45.77\%$), be predicted by measuring the density above the steel in the plastic condition. The prediction equation is:

$$DB = 4.14\ SL - 0.717\ DP + 0.004\ CS + 0.00077\ FI + 220.35 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where:

DB = density below steel (pcf) ; SL = slump (ins)

DP = plastic density (pcf)

CS = 28-day compressive strength (psi)

FI = input frequency (cpm)

This is essentially the same as Equation 4 (page 95), except that in Equation 4 the measured density is in the cured state (DA) while in Equation 10 density is measured in the plastic state (DP).

Since the ANOVA showed that there was no significant difference between these two densities, at the 5% level, one can be substituted for the other in an anlysis. However, it is felt that these equations do not adequately explain variations in the densities below steel. It is hoped that future research will include other parameters which will increase the R^2 term in the analysis of variance, thereby facilitating the identification of other significant terms. Ultimately, it might be possible to measure the slump of concrete, the frequency of vibration and the plastic density; insert a laboratory determined 28-day compressive strength, or a predetermined strength from cores taken from specimens vibrated at stated frequencies; and so compute the density below steel. This computed result can then be checked against the specified density range to see whether the concrete is adequately compacted. If it is not, the inspector can advise the contractor to adjust either his paving speed, his frequency of vibration, or make any other modifications necessary for achieving the required compaction.

The equations presented here are not the answers to all of the vibration problems now encountered in the field. They are merely a guide for future study and research. They may change with different mix designs, with different brands of vibrators, with different eccentric weights in the vibrator and with several other factors that could not be investigated in this experiment. The work herein reported is merely a small step in the direction of furthering our rational knowledge of concrete vibration.

The most effective compaction effort was found to occur at a frequency of 12,000 cpm applied for 7 seconds duration. This confirms

the findings of previous investigators such as Bergstrom and Venkatramiah, who found that most compaction occured within 10 seconds of vibration. This is of interest to the users of slipform pavers since it suggests that good compaction can be achieved at moderate speeds, such as between 10 to 20 fpm. This was also confirmed by Bower and Gerhardt

The measurement of density in the field is still a major problem. The field superintendent wishes to know, immediately after concrete placement, whether the applied frequency is producing an adequate compaction in the concrete pavement. The nuclear density gauge now widely used in the field can only measure densities in the top 2 to 3 inches of a freshly placed concrete pavement. The density at appreciably lower depths beyond the range of the nuclear gauge can only be speculative. As reported in the literature review and shown in Figure 1, slab deterioration will frequently occur below the steel at a depth where nuclear density device is inapplicable and is incapable of producing reliable density readings.

The pulse velocity device seemed impractical for field usage on highway pavements because of the low thickness to width ratio. Pavements are usually constructed between 12 and 48 ft wide but only 8 or 9 inches deep. Even though the equipment can be used with both transducers facing downwards for velocity measurements, the low thickness to width ratio limits the sensitivity of the device to low depths as in the case of the nuclear density device. Furthermore, the device seemed insensitive to minor variations in density and could not be used on plastic concrete. This insensitivity, as mentioned on page 40, was observed before the concrete slump range was changed from

between 1 and 2 inches to between 0 and 1 inches. It appears that there is a need for an equipment which will rapidly measure concrete density in the field, will have a greater range of penetration ability than the presently available nuclear device, and will be adaptable for use on plastic concrete.

RECOMMENDATIONS

Several of the findings of this research have corroborated those of Sullivan (12), Las Forssblad (10), Kolek (8), Bower and Gerhardt (27), and others in terms of the effects of slump, frequency, acceleration and amplitude. This corroboration exists even though different brands of vibrators, different modes of vibration, and different types of mix designs were used in the various experiments.

It is suggested that the findings of this laboratory study be further examined through a field study. It is also suggested that future research might investigate such parameters as paving speeds, air content in concrete, energy generated by vibrators in concrete and its rate of dissipation, and possibly the pressure generated at the base of the concrete during vibration. Studies of these and other parameters should help to explain the variability in the densities measured in concrete.

Based on the findings of this research, the following specific recommendations are offered:

1. The permissible range of slump should be clearly stated. A slump range of between 1 and $2\frac{1}{4}$ inches, as is now specified by many state highway commissions (see ref. 5), seems necessary if proper compaction is to be achieved. This conclusion has been substantiated for the mix designs investigated in this study.

2. The permissible ranges of input frequency and of input amplitude for field vibration of concrete in slipform paving should also be clearly stated. In the opinion of the author, a range of frequency between 10,800 and 12,000 cpm, and an amplitude range of between 0.0045 and 0.0055 inches will be satisfactory for slipform paving if the concrete to be compacted has a slump within the recommended range of 1 to 2¼ inches.
3. There is a need for instrumentation which will indicate to the slipform operator whether vibrators are operating and the frequency or frequency range at which each vibrator is operating. There is also a need for instrumentation whereby, with non-destructive tests, the compaction of concrete can be measured reliably at depths more than 2 or 3 inches below the surface of the slab.
4. Investigation should be initiated to determine what is happening at the interface of the concrete and the vibrator. This knowledge might explain the phenomenon behind the reduction of the input frequency in concrete.

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LIST OF REFERENCES

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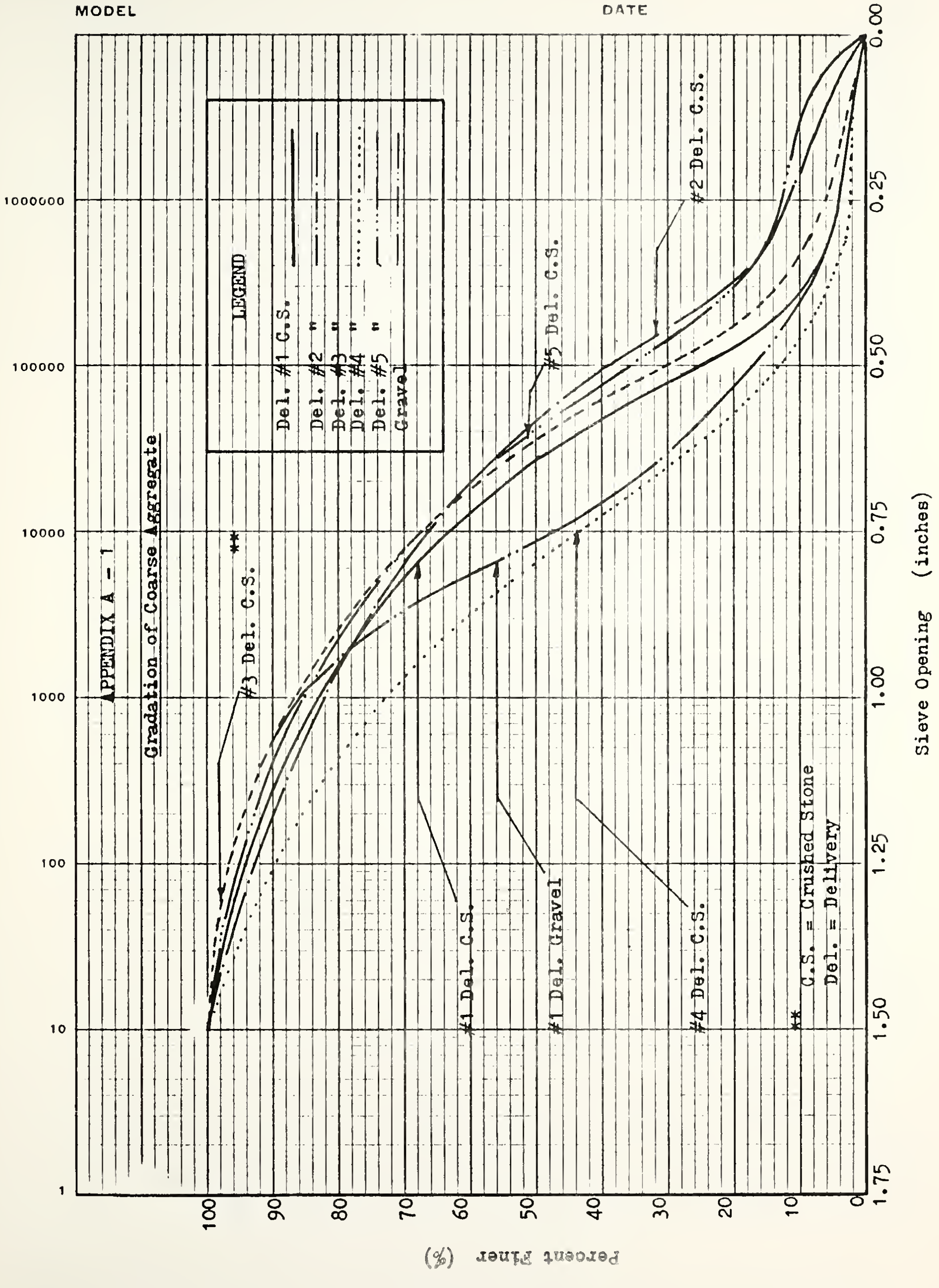
APPENDICES

APPENDIX A

GRADATION CURVES FOR AGGREGATES



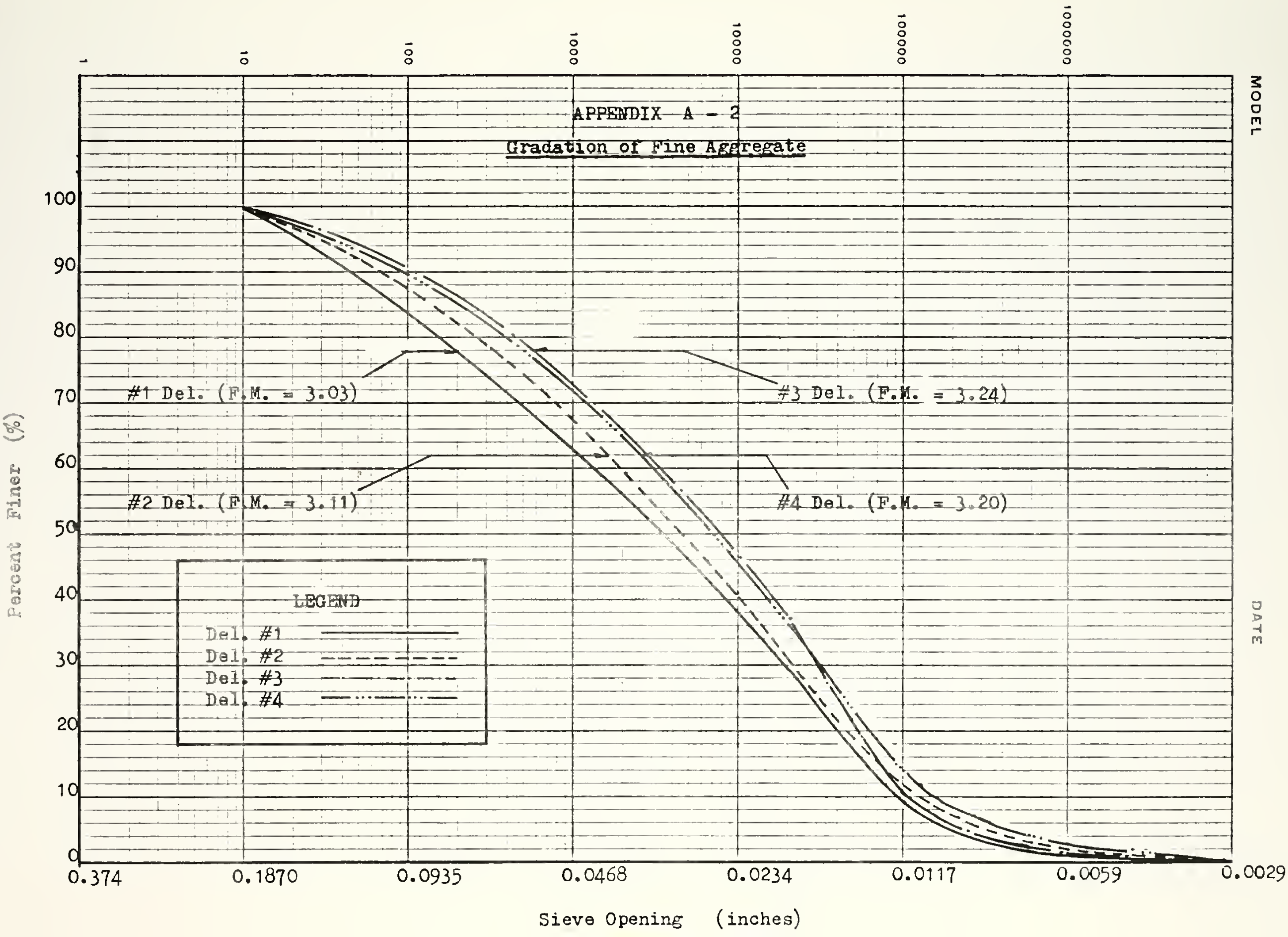
APPENDIX A - 1

Gradation of Coarse Aggregate

KEUFFEL & ESSER CO.

APPENDIX A - 2

Gradation of Fine Aggregate



Pages 117 to 203

The following appendices are not included in this copy of the report for reasons of brevity.

They will be supplied upon request from a recipient of this report. Please state which appendix you are interested in receiving.

Pg

Appendix B: Computer Program for the Analysis of Nuclear Density Data 117

Appendix C: Computer Program for the Analysis of Pulse Velocity Data 124

Appendix D: Computer Program for the Analysis of Acceleration and Displacement Data 128

Appendix E: Data Summary Program 158

Appendix F: Output for Nuclear Density from ANOVA Program (BMD8V) 165

Appendix G: Copies of ANOVA Programs Used 166

Appendix H: Sample Calculation for Pooling 191

Appendix J: Computer Program and Typical Printout for Newman-Keul's Test 192

Appendix K: Typical Printout from Regression Analysis Program "SPSS 15" 201